Decentralized Dynamic Power Control for Cellular CDMA Systems

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***Abstract*—The control of transmit power has been recognized as an essential requirement in the design of cellular code-division multiple-access (CDMA) systems. Indeed, power control allows for mobile users to share radio resources equitably and efficiently in a multicell environment. Much of the work on power control for CDMA systems found in the literature assumes a quasi-static channel model, i.e., the channel gains of the users are assumed to be constant over a sufficiently long period of time for the control algorithm to converge. In this paper, the design of dynamic power control algorithms for CDMA systems is considered without the quasi-static channel restriction. The design problem is posed as a tradeoff between the desire for users to maximize their individual quality of service and the need to minimize interference to other users. The dynamic nature of the wireless channel for mobile users is incorporated in the problem definition. Based on a cost minimization framework, an optimal multiuser solution is derived. The multiuser solution is shown to decouple, and effectively converge, to a single-user solution in the large system asymptote, where the number of users and the spreading factor both go to infinity with their ratio kept constant. In a numerical study, the performance of a simple threshold policy is shown to be near that of the optimal single-user policy. This offers support to the threshold decision rules that are employed in current cellular CDMA systems.**

***Index Terms*—Code-division multiple access (CDMA), dynamic programming, optimal control, power control, spread spectrum, wireless communication.**

I. Introduction

In a cellular code-division multiple-access (CDMA) environment, a set of mobile users shares a common bandwidth allocation. The burden of allocating radio resources equitably on the reverse link is handed over to a power control algorithm. In this paper, we consider the control of transmit power for CDMA systems in the context of constant rate applications. By constant rate applications, we refer to real-time applications with stringent delay constraints and fixed data rate requirements such as cellular telephony. For any such application, an appropriate wireless connection must be maintained at all times. The alternative of buffering data and scheduling users based on their link quality is not acceptable because of the inherent delay associated with this paradigm. We therefore center our attention on power control algorithms that attempt to provide users with a uniform quality of service over time.

Previous work on power control for cellular systems includes the study of optimal transmission strategies [1]–[7], as well as the design of practical power control policies [8], [9]. Power control has been shown to increase the bandwidth efficiency and the capacity of both channelized cellular systems [10], [11] and cellular CDMA systems [12]–[14]. Early analytical work [2], [11], [15] has focused on maximizing the minimum user signal-to-interference ratio (SIR), an approach known as SIR balancing. The problem of power control has subsequently been redefined as minimizing the total transmitted uplink energy subject to maintaining the SIR of each user above an individual threshold value [3], [5]. The latter formulation better incorporates the notion of quality of service and it is suitable for heterogeneous systems, since it allows for the specification of individual link requirements. Distributed power control algorithms have been studied in a wide range of contexts [1], [4], [6], [8]. Their implementation usually entails iterative methods. An elegant framework which provides insightful results about iterative power control algorithms for quasi-static channels has been proposed by Yates [5]. Integrated power control and base station assignment has been investigated by Hanly [16], and Yates and Huang [17]. Hanly [7] has also addressed the joint topic of power control and capacity for spread spectrum systems.

Most of the work found in the literature on this topic [1]–[11], [15], [17]–[20] has been restricted to quasi-static channel models, i.e., models in which the channel gain of every user is assumed to remain approximately constant over sufficiently long periods of time. The performance results obtained under this assumption will be valid so long as the reaction time of the power control algorithm is small compared with the coherence time of the underlying wireless channel. In other words, the transmit power of each user is implicitly assumed to converge to its optimal level before any significant change occurs in the channel state. Unfortunately, this assumption may not necessarily hold for practical systems [18].

A second set of papers on power control [21]–[24] recognizes the random nature of the wireless link. Yet, in these papers, the controller is assumed to have instantaneous knowledge of the current channel state. In other words, the controller knows the exact state of the channel at the beginning of the transmission interval. Under this assumption, the optimal power assignment depends solely on the probability density function of the channel gain, and not on the time dynamics of the channel evolution. Although such a formulation provides insightful results on the capacity of fading channels, it does not account for feedback delay or for imperfect state estimates, for instance.

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The need to incorporate the stochastic, dynamic nature of the wireless channel into the power control problem formulation has been identified clearly in previous work [18], [24]. In this paper, we follow this avenue of research and model the wireless channel as a stochastic process. Furthermore, we acknowledge that the controller has to select an appropriate transmission level at the beginning of the control period and that it must therefore rely on the past values of the channel state to predict the current one. This contrasts with the quasi-static power control problem formulation, or "snap-shot" approach, where the channel gain of every user is assumed to remain constant. More specifically, we include a stochastic channel model as part of the problem definition. This strategy enables us to account for channel dynamics in the derivation of an optimal solution rather than relying primarily on the algorithm structure for adaptation. We emphasize however that an efficient power control algorithm can only be derived if the time over which the channel gain is highly correlated is much greater than the time period between successive control signals. This leads us to define a system parameter, called the control period T_a, as the reciprocal of the control rate f_a, i.e.,

$$T_a = \frac{1}{f_a}. \tag{1}$$

The smaller the value of T_a, the more rapidly the power control algorithm can adapt to a change in channel gain. In addition to a small control period, a realistic control policy must account for physical constraints present in the system. One such restriction is a limited data rate dedicated to power control on the forward link. The need to adapt promptly to channel variations together with a constrained feedback data rate point to power control algorithms that use simple, frequent control signals. In this work, our goal will be to design a pragmatic uplink power control algorithm in context of constant rate applications. We pose the design problem as a tradeoff between the desire for users to maximize their individual quality of service and the need to minimize interference to other users. The corresponding decision rule should account for the dynamic nature of wireless channels and the limited feedback rate on the forward link.

To account for the stochastic nature of the wireless channel, we cast the power control problem in a Markov decision process framework. Once this is established, we characterize the behavior of the optimal solution in the large system asymptote, where the number of users and the spreading factor both go to infinity with their ratio kept constant. We show that, under certain conditions, the multiuser power control problem effectively decouples into a set of single-user power allocation problems. An approximate solution for the single-user problem is then proposed, and its performance is compared with that of the optimal single-user solution.

The remainder of this paper is organized as follows. In Section II, we introduce a mathematical model for the reverse link and we discuss some of the issues involved in controlling a cellular network. We pose the power control problem formally in Section III. We present the structure of an optimal power control policy in Section IV, and provide numerical examples to illustrate the behavior of a controlled system in Section V. We give our conclusions in Section VI.

II. System Model

We begin with the development of a stochastic model for the wireless system we wish to study. To facilitate tractability, we would like the channel gains to be independent and identically distributed (i.i.d.) random processes across users. Fortunately, this is readily obtained by assuming that the users move around the cell independently, a reasonable assumption. Hereafter, we adopt this premise and embrace its simplicity. We discuss the single-user channel first with the understanding that the multiuser system is an aggregate of independent single-user channels.

A. Single-User System

The system model we consider for the single-user wireless connection pertains to the class of stationary discrete-time Markov decision processes. We review briefly the specifics of Markov decision processes and establish a convenient notation for the power control problem at hand. We will show in Example 1 how this general framework can accommodate a Rayleigh-fading channel with a first-order autoregressive autocorrelation function.

A Markov decision process is characterized by a control transition probability P_a, which we define via

$$\begin{aligned} P_a(x, y) &\triangleq \mathrm{Prob}\left\{x_{t+1} = y \,|\, x_t = x,\ a_t = a\right\} \\ &= \mathrm{Prob}\left\{x_1 = y \,|\, x_0 = x,\ a_0 = a\right\}. \end{aligned} \tag{2}$$

For all discrete time t, the state x_t is an element of a space $\mathcal{X}$, while the action a_t is an element of a space $\mathcal{A}$. The action a_t is further constrained to take values in a nonempty subset $\mathcal{A}(x)$ of $\mathcal{A}$ whenever $x_t = x$. Our initial endeavor will be to define the *state space* $\mathcal{X}$ and the *action space* $\mathcal{A}$ for the control problem at hand.

We construct the state space $\mathcal{X}$ by describing each of its components. Let $\mathcal{S}$ be the set of all possible channel gains, which we refer to as the *channel state space*. We assume that $\mathcal{S}$ is finite and use

$$0 \le s^{(1)} < s^{(2)} < \cdots < s^{(|\mathcal{S}|)}$$

to denote its members. This assumption is not too restrictive since successive refinements of the channel state space allow for arbitrary precision. Furthermore, the use of numerical methods in practical systems requires that $\mathcal{S}$ be approximated by a finite set. The corresponding wireless channel is a Markov sequence with probability transition matrix Q

$$Q(s, z) \triangleq \mathrm{Prob}\left\{s_{t+1} = z \,|\, s_t = s\right\}.$$

Similarly, we define $\mathcal{P}$ to be the collection of all possible power levels at which a mobile unit can communicate. Given a limited feedback bandwidth, we feel justified in making the *transmit power space* $\mathcal{P}$ finite. We write the power levels of $\mathcal{P}$ as

$$0 \le p^{(1)} < p^{(2)} < \cdots < p^{(|\mathcal{P}|)}.$$

For channel gain $s \in \mathcal{S}$ and transmit power $p \in \mathcal{P}$, the power received at the base station, which we represent by r, is given by

$$r = sp.$$

The state space $\mathcal{X}$ for the single-user system is the set product of the channel space and the transmit power space

$$\mathcal{X} = \mathcal{S} \times \mathcal{P}.$$

We represent the elements of $\mathcal{X}$ by two dimensional vectors of the form $x = (s, p)$, where $s \in \mathcal{S}$ and $p \in \mathcal{P}$. Also, we employ the indexed notation $x_t = (s_t, p_t)$ for the state of the system at discrete time t. Appending the transmit power to the channel state to create the system space $\mathcal{X}$ allows us to consider a large class of control policies. This will be made obvious in what follows.

Turning to the construction of the action space $\mathcal{A}$, we note that the transmit power level is our sole input to the system. We thus make the obvious choice of equating $\mathcal{A}$ to the transmit power space $\mathcal{P}$. This leads to the natural evolution equation

$$p_{n+1} = a_n.$$

Hence, the controller decides at which level the mobile unit transmits next. The channel probability transition matrix and the control transition probability are related through the equation

$$P_a\left((s, p), (z, q)\right) = Q(s, z)\mathbb{1}_{\{q=a\}} \tag{3}$$

where $\mathbb{1}_B$ is the indicator function for event B. Equation (3) asserts that the evolution of the channel gain is independent of the power level at which the mobile unit is transmitting. In fact, the state transition probability is obtained directly from the channel transition probability $Q(s, z)$ provided that q is equal to a, the power level specified by the controller; otherwise, the transition probability is zero.

A more delicate problem lies with the selection of the constraint subsets $\{\mathcal{A}(x)\}$. While choosing $\mathcal{A}(x) = \mathcal{A}$ for all states $x \in \mathcal{X}$ offers the greatest flexibility, it also requires more feedback bandwidth than, say, simple binary up/down control. For binary power control, the family of action subsets corresponds to

$$\begin{aligned} \mathcal{A}_{\text{bin}}(x) &= \mathcal{A}_{\text{bin}}\left(\left(s, p^{(j)}\right)\right) \\ &= \left\{p^{(\max\{1, j-1\})}, p^{(\min\{j+1, |\mathcal{P}|\})}\right\}. \end{aligned} \tag{4}$$

Fortunately, a specific choice of constraint subsets need not be made for our later results to hold. We do however impose some mild conditions on the collection $\{\mathcal{A}(x)\}$. We assume that $\mathcal{A}((s, p))$ depends exclusively on p, the transmit power, and we want any transmit power level $p^{(j)}$ to be reachable from any other transmit power level $p^{(i)}$ within finite time.

Remark 1: In our definition, the wireless channel sequence forms a Markov chain. We could have equivalently considered a Markov-m random sequence in which the state of the channel at time t depends on its m previous values. However, for ease of notation, we retain the use of a simple Markov chain with the understanding that this work can readily be extended to the more general channel model.

Example 1: To illustrate how the Markov decision framework can accommodate familiar channel models, we consider a Rayleigh-fading environment. We model the baseband channel gain as a first-order autoregressive random process. The innovation process is a sequence of i.i.d. zero-mean circularly complex Gaussian random variables and the evolution of the baseband complex channel follows a simple linear recursion. For complex gain e and autocorrelation coefficient ρ, the evolution equation is equal to

$$e_{t+1} = \rho\, e_t + w_t \qquad t = 0, 1, 2, \ldots \tag{5}$$

where $\{w_t\}$ is a sequence of i.i.d. zero-mean circularly complex Gaussian random variables. The channel power gain s is given by

$$s_t = |e_t|^2 = (\mathrm{Re}(e_t))^2 + (\mathrm{Im}(e_t))^2.$$

We observe that the gain sequence $\{s_t\}$ also forms a Markov process. More precisely, s_t conditioned on $\{s_\tau; \tau < t\}$ has a noncentral chi-squared distribution with two degrees of freedom and noncentrality parameter

$$\delta_t^2 = \rho^2\, s_{t-1}^2.$$

It follows from (5) that the first-order statistic of s_t is an exponential distribution, the distribution corresponding to a Rayleigh-fading model. To obtain a finite-state Markov chain that mimics this channel, it suffices to partition the range of the channel power gain s and to derive a probability transition matrix $Q(s, z)$ based on the corresponding noncentral chi-squared distributions. The resulting system is inherently memoryless, and successive refinements of the partition leads to arbitrary precision in the stationary distribution of the Markov chain.

B. Multiuser System

Since we assume the channel gains of all the users to be i.i.d. random processes, the multiuser system is characterized completely by the single-user model of Section II-A. Here, we make this extension explicit and discuss some of the ideas involved in lifting the single-user system to its multiuser counterpart.

For a system with K users, the augmented system space $\mathcal{X}^K$ is the set product of K copies of the single-user state space $\mathcal{X}$. Its elements may be viewed as row vectors

$$\mathbf{x} = (x(1), x(2), \ldots, x(K))$$

where $x(k) \in \mathcal{X}$ for $k = 1, 2, \ldots, K$. Similarly, the multiuser action space $\mathcal{A}^K$ is the set product of K copies of the single-user action space $\mathcal{A}$ with controls

$$\mathbf{a} = (a(1), a(2), \ldots, a(K))$$

where $a(k) \in \mathcal{A}$ for $k = 1, 2, \ldots, K$. The wireless channels being independent across users, the probability of an event in the augmented system is given by the product of the probabilities of its components. The control transition probability of the augmented Markov chain is, therefore, obtained through the formula

$$P_{\mathbf{a}}(\mathbf{x}, \mathbf{y}) = \prod_{k=1}^{K} P_{a(k)}\left(x(k), y(k)\right)$$

where P_a is the single-user control transition probability of (3).

Again, the action $\mathbf{a}$ is constrained to take values in a nonempty subset $\mathcal{A}^K(\mathbf{x})$ of $\mathcal{A}^K$. For the multiuser system, we only admit actions of the form

$$\mathcal{A}^K(\mathbf{x}) = \mathcal{A}(x(1)) \times \mathcal{A}(x(2)) \times \cdots \times \mathcal{A}(x(K)). \quad (6)$$

Although the spaces $\mathcal{X}^K$ and $\mathcal{A}^K$ arise naturally as extensions of the single-user model, condition (6) does not. Nonetheless, we claim that all the constraint families of interest are of this form and we make this condition a constituent of our multiuser system model. The basic implication of condition (6) is that the set of power levels at which a mobile unit can transmit next depends exclusively on the power level at which it is currently transmitting, i.e., the set of admissible power levels does not depend on the transmit power of other users. This is quite reasonable; for instance, in binary up/down power control, a mobile unit can only update its power to a level adjacent to its current transmit power level.

III. Problem Formulation

For CDMA systems, power control on the reverse link breaks down into two components, initialization and tracking. The initialization procedure is straightforward. It involves taking measurements of a pilot signal strength, and then determining the initial transmission power level by inverting the estimated channel (see, e.g., [12]). The second component of power control, tracking, is more involved as it requires a sophisticated channel model and a precise definition of quality of service. In this paper, we focus entirely on the latter aspect of power control.

For multicell environments, a power control algorithm in which each cell operates independently is desirable. The alternative of coordinated multicell power control involves added infrastructure for data transmission, computation complexity, and delay. Thus, we would like the power of mobile users to be controllable on a single-cell basis. In words, we want each cell to form an autonomous entity, whereby the base station controls the power of incell users based exclusively on local measurements. Yet, we need to account for intercell interference in our model. This leads us to the following assumption.

Assumption 1 (Single-Cell Framework): The interference contribution from neighboring cells is a white Gaussian process. This random process is independent of background noise and its power spectral density is proportional to the power spectral density of the background noise.

The Gaussian assumption is not surprising. Indeed, for large cellular CDMA networks the output interference of a linear receiver can be well approximated by a Gaussian process [25]. In contrast, the claim that the underlying Gaussian process has constant power spectral density is harder to motivate. Typically, intercell interference will be dominated by a few immediate neighbors (e.g., six neighbors for hexagonal cells). Unless the interference power spectral density generated by these neighbors is itself constant, there will be fluctuations in the interference level seen by the base station of interest. To ensure coherence of our framework, we will control the variance of the power spectral density produced by each cell. This in turn will limit fluctuations in interference level across cells, validating Assumption 1 over time frames of interest.

We cast the multiuser power control problem in a cost minimization framework. From the discussion above, we gather that the two elements to be minimized are the sum of the received powers

$$S \triangleq \sum_{k=1}^{K} r(k)$$

and some cost function which provides a figure of merit for the quality of a wireless connection. We now elaborate on this cost function, which we denote by g. The performance of a CDMA communication system with a matched filter receiver front-end can be expressed in terms of SIR at the output of the receiver. This statement is certainly true whenever the system is supporting a large number of users and the signal of each user is decoded independently. In such a case, the output of the linear receiver approaches a Gaussian random process and the associated SIR accurately predicts the bit-error rate at the output of the decoder, which in turn yields a measure of user satisfaction. For matched filter receivers, the SIR at the input of the decoder can be computed from the received power $r(k)$ and the total incell interference $\sum_{\ell \neq k} r(\ell) = S - r(k)$. More specifically, we have

$$\gamma(k) = \frac{N r(k)}{(1+\alpha) N_0 W + S - r(k)} \quad (7)$$

where N is the spreading factor, N_0 is the power spectral density of the background noise, W is the system bandwidth, and α is the proportionality constant corresponding to out-of-cell interference as defined in Assumption 1. For received power $r(k)$ and total incell interference $S - r(k)$, the cost function g is equal to

$$g(\gamma(k)) = g\left(\frac{N r(k)}{(1+\alpha) N_0 W + S - r(k)}\right). \quad (8)$$

Possible candidates for g may be derived from the achievable information rate for a given probability of error. However, we emphasize that constant rate applications, which are the focus of this paper, are not resource greedy in the sense that they do not benefit much from exceedingly good connections. The performance metric g should then feature diminishing returns, for otherwise the base station might try to deliver superfluous channel capacity to incell users at the expense of excessive transmit energy and increased interference. Without specifically choosing a performance metric, we assume that g belongs to the class of bounded continuous functions. Also, since γ is monotone increasing in r, the penalty function g should be monotone decreasing in r; a property that we take for granted.

Remark 2: In the discussion above, performance is characterize in terms of SIR. A more fundamental measure for the performance of a communication system is the energy per information bit divided by the total interference density at the input of the receiver. The latter metric characterizes coded and uncoded systems alike, and is given by

$$\tilde{\gamma}(k) = \frac{\Omega r(k)}{(1+\alpha) N_0 W + S - r(k)}$$

where Ω is the bandwidth expansion factor (see, e.g., [26]). We note that the results contained in this paper hold verbatim if we replace N by Ω and γ by $\tilde{\gamma}$. However, since SIR is much more common to the communication literature, we retain its use in (7) for the sake of clarity.

Let an admissible *stationary Markov policy* be a mapping μ from the state space $\mathcal{X}^K$ to the action space $\mathcal{A}^K$ such that $\mu(\mathbf{x}) \in \mathcal{A}^K(\mathbf{x})$ for all $\mathbf{x} \in \mathcal{X}^K$. An optimal power control algorithm is an admissible policy μ that provides the best tradeoff between $\mathrm{E}[S]$ and $\mathrm{E}\left[\sum_{k=1}^{K} g(\gamma(k))\right]$. In this work, we adopt a Bayesian formulation for the tradeoff problem; Bayesian frameworks are common in problems of optimal control [27], and detection and estimation theory [28]. We pose the control problem we wish to solve as follows.

Problem 1: Find an admissible Markov policy μ that minimizes the cost function

$$J(\mu) \triangleq \frac{1}{K}\left(\lambda \mathrm{E}[S] + \mathrm{E}\left[\sum_{k=1}^{K} g(\gamma(k))\right]\right) \tag{9}$$

over the set of all stationary Markov control policies, where λ in (9) is a tradeoff parameter.

Under policy μ, the control system evolves according to the transition probability

$$P_\mu(\mathbf{x}, \mathbf{y}) = \mathrm{Prob}\left\{\mathbf{x}_{t+1} = \mathbf{y} \,|\, \mathbf{x}_t = \mathbf{x},\ \mathbf{a} = \mu(\mathbf{x})\right\}.$$

The tradeoff parameter λ may be interpreted as the relative cost of excess power versus poor quality of service. We note that (9) gives a per user figure of merit for system performance. A normalized metric is convenient as it permits an easy comparison of systems with different numbers of users.

IV. Optimal Power Control Policy

Most of the results we present in this section are asymptotic in nature. We consider the limiting regime, where the number of incell users goes to infinity. To accommodate this increasing number of users, it is reasonable to assume that the system bandwidth and the spreading factor are also increasing. We refer the reader to Verdú and Shamai [29], and Tse and Hanly [30] for a detailed discussion of large system analysis in context of multiuser CDMA systems.

Definition 1 (Large System Asymptote): Let the loading factor b be the ratio of the number of users to the spreading factor

$$b \triangleq \frac{K}{N}.$$

We define the *large system asymptote* as the limiting regime where the number of users and the spreading factor both go to infinity with their ratio kept constant at b.

From this point on, the reader should recognize that, in the asymptotic regime, we first fix the loading factor and then let the number of users go to infinity. In particular, the expression $K \to \infty$ implies that $N \to \infty$ with $b = K/N$ fixed. Furthermore, since we will be considering systems with varying number of users, we note that the domain and the range of the policy μ depend on the number of users K. Throughout, we leave this dependence implicit as this should cause no confusion.

In Section III, we expressed the importance of controlling the variance of the total received power for the single-cell framework to be valid. The underlying assumption is that, for a given policy, the interference contribution from a given cell to its neighbors is proportional to the total power received from the incell users. To better illustrate the need to control the variance of the received powers, we consider a cellular environment with seven hexagonal cells: a central cell and six peripheral neighbors. An approximate expression for the SIR of the users in the central cell is given by

$$\gamma(k) = \frac{N r(k)}{N_0 W + l D^{-4} \sum_{j=1}^{6} S^{(j)} + S - r(k)} \tag{10}$$

where D is the distance between two neighboring base stations, and $S^{(j)}$ is the total incell power received at the base station of cell j. The path loss model in (10) is $L(d) = l d^{-4}$, where l is some fixed constant. Clearly, a power control algorithm that produces small variations in the $S^{(j)}$ also yields an interference level which is approximately constant, as desired.

To control fluctuations in total received power, we restrict the set of policies over which the minimization in Problem 1 is taken. We note that any stationary Markov policy μ induces a finite state Markov chain on the state space $\mathcal{X}^K$. With a fully connected channel transition matrix, any nontrivial stationary Markov policy induces a unique stationary distribution on $\mathcal{X}^K$. For any such policy μ, the corresponding variance in received power, $\mathrm{Var}[S]$, is well defined. Being primarily concerned with large systems, and looking at (10), we gather that $\mathrm{Var}[S] = o(K^2)$ is a sufficient condition for the single-cell framework to hold in the large system asymptote of Definition 1. More specifically, we say $\mathrm{Var}[S] = o(K^2)$ as $K \to \infty$ whenever

$$\mathrm{Var}\left[\frac{S}{K}\right] \to 0$$

as $K \to \infty$. We choose to restrict our search space to stationary Markov policies for which

$$\mathrm{Var}[S] < KB$$

where B is some (large) constant. This condition is sufficient to ensure coherence of the single-cell framework in the limiting regime of Definition 1.

Also, because we are interested in power control and not in admission control, the power control policy we adopt should not be selective across users. That is, the expected quality of service should be identical for all users, as opposed to purposely dropping one user to the benefit of others. We therefore further restrict our search space to nonselective Markov policies. We denote by Π the set of all nonselective stationary Markov policies for which $\mathrm{Var}[S] < KB$

$$\Pi = \left\{\mu\colon \begin{array}{l} \mathrm{E}[g(\gamma(k))] = J(\mu) - \dfrac{\lambda}{K}\mathrm{E}[S]\ \forall k \\ \mathrm{Var}[S] < KB \end{array}\right\}.$$

The modified power control problem can then be posed as follows.

Problem 2: For each K, find an admissible Markov policy $\mu \in \Pi$ that minimizes the cost function

$$J(\mu) \triangleq \frac{1}{K}\left(\lambda \mathrm{E}[S] + \mathrm{E}\left[\sum_{k=1}^{K} g(\gamma(k))\right]\right) \tag{11}$$

over the set of all stationary Markov control policies in Π.

Admittedly, finding an optimal solution $\mu^* \in \Pi$ to the power control problem is no simple task. We therefore consider conditions under which the multiuser power control problem decouples, and effectively converges to a collection of independent single-user systems. Decoupling is beneficial since it greatly reduces the dimension of the search space for an optimal policy. Furthermore, it leads to the desirable distributed property by which the transmit powers of the mobiles are controlled independently. In particular, we characterize the behavior of a cellular network in the large system asymptote of Definition 1. First, we state a simple consequence of the fact that we only admit policies in Π.

Corollary 1: Let $\{\mu_1, \mu_2, \ldots\}$ be a sequence of admissible policies such that $\mu_K \in \Pi$ is a policy for a system with K users then

$$\lim_{K\to\infty} \mathrm{Var}\left[\frac{S}{K}\right] = 0. \tag{12}$$

Corollary 1 provides firm ground for Assumption 1. Indeed, this corollary implies that $\mathrm{Var}[S/N]$ also vanishes for sufficiently large systems. We can, therefore, safely assume that the interference contribution from other cells remains constant. To reduce the complexity of the controlled system, we would like to find an optimal single-user policy. That is, a policy in which decisions concerning the control of a particular user is based strictly on its transmit power p, its channel gain s, and the slowly varying total received energy S. For systems with more than one user, a single-user policy could be far from optimal. Fortunately, this is not the case for sufficiently large systems, since for such systems there exist single-user policies which are ϵ-optimal as seen in the following result whose proof is given in the Appendix.

Theorem 1: Suppose we fix the loading factor b. Given any $\epsilon > 0$, there exists an integer M such that $K > M$ implies the existence of a stationary Markov policy $\hat{\mu}$ for which

$$J(\hat{\mu}) < J(\mu^*) + \epsilon \tag{13}$$

where μ^* is an optimal policy, $\hat{\mu}$ is a randomized single-user policy of the form

$$\hat{\mu}(\mathbf{x}) = (u(x(1)),\, u(x(2)),\, \ldots,\, u(x(K)))$$

and u is an admissible control policy for the single-user Markov decision process described at the beginning of Section II.

This theorem asserts that for large systems we can trade the very complex multiuser policy μ^* for a simple, easy to implement randomized single-user policy $\hat{\mu}$ without compromising system performance. Because of the complexity of multiuser policies, it is not possible to determine numerically how large K should be for this asymptotic behavior to come into play. However, the convergence of $\mathrm{Var}[S/N]$ seems to depend on the ability of the power control algorithm to even out variations in received power. In the performance analysis of linear multiuser detectors for CDMA systems, it has been observed that the large system asymptotics for the SIR are valid for very moderate values of K and N of the order of 50 (see, e.g., [29], [30]). Based on these results, we expect the performance of the optimal single-user policy to match that of the optimal multiuser policy very rapidly as K increases.

In view of Proposition 1, and for large systems, there is no loss in optimality in considering single-user policies. We henceforth focus on such policies for which users are controlled independently. Then, the power control algorithm is a state feedback law whose sole input is the current state of the user of interest. It follows from the law of large numbers that for single-user policies, variance $\mathrm{Var}[S/K]$ vanishes in the large system asymptote. Incidentally, $\mathrm{Var}[S/N]$ also vanishes and the interference contribution from all incell users can be assumed constant. Thus, unlike in the case of centralized policies, we need not worry about controlling variations in intercell interference. Furthermore, because the normalized variance $\mathrm{Var}[S/N]$ goes to zero for large systems, we can assume that the incell interference power remains approximately constant over time. For convenience, we denote this constant incell interference power spectral density by I_0.

A. Single-User Policy

The single-user policy of Proposition 1 is a randomized policy. That is, for any given state $x \in \mathcal{X}$, the action $u(x)$ applied by the controller is selected at random according to a state dependent probability distribution on the action space $\mathcal{A}(x)$. In practical systems, however, a deterministic control policy may be more appropriate. Indeed the complexity of a randomized single-user policy, or the complexity of a multiuser policy for that matter, may be overwhelming. Optimal deterministic policies can be computed efficiently using standard tools from dynamic programming such as value iteration, policy iteration, or linear programming. The reader is referred to Bertsekas [31] for an in depth treatment of dynamic programming and optimal control. Here, we assume that the reader is familiar with dynamic programming and we proceed to cast the power control problem in a dynamic programming framework. We write the single-user cost per stage function as

$$c(x) \triangleq \lambda r + g\left(\frac{Nr}{(1+\alpha)N_0W + I_0W - r}\right)$$

and we evaluate Markov policies based upon the average cost criterion

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} c(x_t) \tag{14}$$

where the single-user system evolves according to the Markov transition probability

$$P_u(x,y) = \mathrm{Prob}\left\{x_{t+1} = y \,|\, x_t = x,\ a_t = u(x)\right\}.$$

This is a standard formulation in optimal control and it falls in the class of infinite horizon average cost problems. We note that for any single-user policy of the form $\hat{\mu}(\mathbf{x}) = (u(x(1)),\, u(x(2)),\, \ldots,\, u(x(K)))$

$$\begin{aligned} J(\hat{\mu}) &= \frac{1}{K}\left(\lambda \mathrm{E}_{\hat{\mu}}[S] + \mathrm{E}_{\hat{\mu}}\left[\sum_{k=1}^{K} g(\gamma(k))\right]\right) \\ &= \lambda \mathrm{E}_u[r] + \mathrm{E}_u[g(\gamma)] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} c(x_t) \end{aligned}$$

by ergodicity of the finite state system. The average cost of (14) is then equivalent to the multiuser average cost of (9) for any such single-user policy.

The discussion above reveals the interplay between three fundamental quantities: the expected quality of service $\mathrm{E}[g(\gamma)]$, the interference to other users $\mathrm{E}[r]$, and the number of incell users

$$K = \left\lfloor \frac{I_0 W}{\mathrm{E}[r]} \right\rfloor.$$

For instance, for a fixed interference power spectral density I_0, the total number of incell users decreases as users demand better connections. For a specific cellular system, the relation between $\mathrm{E}[g(\gamma)]$, $\mathrm{E}[r]$, and K can be made explicit by repetitively solving the dynamic program defined above for various values of λ and I_0. In the next section, we study how this framework can be employed to improve the performance of the power control loop in the IS-95 standards [32].

V. Numerical Example

In this section, we illustrate how dynamic programming applies to power control. The channel power gain of a typical wireless connection varies as the mobile user travels around the cell. Such variations are known to happen on two different scales. Shadow fading is a slowly varying process generated by changes in the environment of the mobile. These changes occur on the scale of distance between objects (such as buildings) in the environment. The second scale of variations, multipath fading, is much smaller as it happens over distances of the order of a wavelength of the carrier. At vehicular speed (velocity less than 100 km/h), the mean path gain and the shadow fading process remain essentially constant over multiple control periods. Multipath variations however do not. In what follows, we assume that the mean path gain and the shadow fading process are approximately constant over time frames of interest. Accordingly, variations in channel gain are entirely due to multipath fading.

To model multipath fading, we revisit the wireless channel introduced in Example 1. More specifically, we consider a first-order autoregressive random process with complex gain e and autocorrelation coefficient ρ, as described in (5). This channel model corresponds to the standard Rayleigh-fading environment [33] with an autoregressive autocorrelation structure. The autocorrelation coefficient ρ may depend on the speed of the mobile v, the nature of the environment, and the wavelength ψ of the carrier. If we consider the situation where the propagation environment is such that power is received uniformly from all directions, the autocorrelation function of the channel can be modeled as a Bessel function of the first kind of order zero [34], i.e.,

$$\mathrm{E}[e_{t+\tau} e_t^*] = J_0\left(\frac{2\pi v \tau}{\psi}\right)$$

where e^* denotes the complex conjugate of e. In general, the channel will be more strongly correlated if the power is not received uniformly from all directions. In our numerical analysis, we consider correlation coefficients of the form

$$\rho = J_0\left(\frac{2\pi v T_a}{\psi}\right).$$

For a carrier frequency of 900 MHz and a control period of 1.25 ms, it is sufficient to consider correlation coefficients from 0.8 to 1 to cover the entire range of reasonable vehicular speeds.

TABLE I
CHANNEL PARAMETERS

$W = 1.25$ MHz	total bandwidth
$R = 14.4$ kbps	bit rate
$N_0 = -169$ dBm/Hz	noise density
$I_0 = -159$ dBm/Hz	interference density
$G = -108$ dB	mean path gain
$\alpha = 6$	intercell factor

The parameter values for the wireless channel used in our numerical results appear in Table I.

In practical systems, the feedback bit rate dedicated to power control information is fixed (e.g., 800 bps for the IS-95 standard [32]); as a consequence, simpler control signals imply more frequent power updates. Having stressed in the introduction how critical a short control period is to high capacity cellular networks, we expect simpler algorithms with frequent updates to outperform more complex power control algorithms. We first study the class of binary up/down power control algorithms. For any given feedback bit rate, these algorithms yield the shortest possible control period. Moreover, they are widely employed in IS-95 based CDMA systems [32]. When proportional increments are used, the family of constrained subsets corresponding to binary control $\{\mathcal{A}_{\mathrm{bin}}(x)\}$ is as described in (4). Furthermore, at each control instant a mobile can either increase or decrease its transmit power by an amount proportional to its current power level

$$p_{t+1} \in \left\{\frac{1}{\beta} p_t,\ \beta p_t\right\}$$

where β is a designated constant; the IS-95 standard suggests a value of $\beta = 1.25$. The numerical results are obtained for a finite channel state space, and the system parameters for our simulations are found in Table II. For a given interference power spectral density I_0, the SIR at the output of the matched filter γ can be expressed in terms of the channel gain s and the transmit power p as

$$\gamma_t = \frac{s_t p_t}{(1+\alpha)\sigma^2 + I_0 W}.$$

The individual cost per stage function was originally introduced in Section III. In our analysis, we take the following cost function:

$$g(\gamma) = \begin{cases} e^{-1}, & \gamma < 3 \\ (\gamma - 2)e^{-(\gamma-2)}, & \gamma \geq 3. \end{cases} \tag{15}$$

We put an upper bound on the penalty g with the understanding that below a certain SIR, the decoder outputs incoherent data. The cost as a function of SIR appears in Fig. 1.

The optimal stationary control policy can be obtained by direct application of either the successive approximation method or the policy iteration algorithm. Both techniques are standard dynamic programming tools. In Fig. 2, we show the optimal decision policy as a function of transmit power p and channel gain s; the correlation coefficient of the simulated channel is $\rho = 0.95$. Unfortunately, the optimal policy has no simple structure and is, therefore, difficult to implement in practical systems. Computing the solution to the dynamic program in real time

TABLE II
SYSTEM PARAMETERS

$\mathcal{P}_{max} = 24$ dBm	maximum radiated power
$\mathcal{P}_{min} = 5$ dBm	minimum radiated power
$T_a = 1.25$ ms	control period
$\beta = 1.25$	proportionality constant

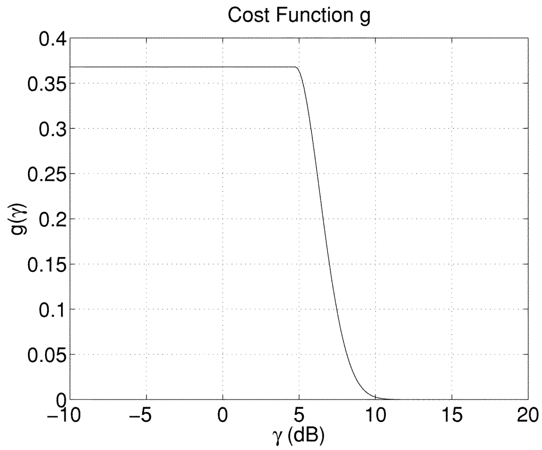

Fig. 1. Plot of the cost metric g as a function of SIR. A low SIR induces a high penalty, while a sufficiently high SIR yields no cost.

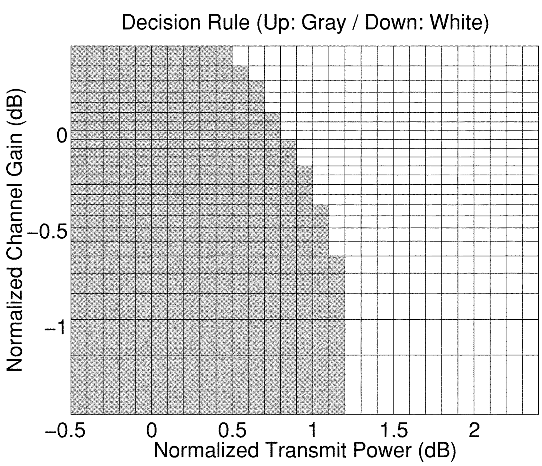

Fig. 2. Graphical display of the optimal binary single-user power control policy. As the link quality improves, the mobile unit reduces its power consumption.

would be too time consuming. To employ this control policy in a real cellular system, the decision rule would have to be stored as a look-up table. Although storing the control policy for one specific channel profile can be achieved rather easily, doing so for every possible combination of system parameters is again impractical. We therefore explore simpler alternatives to the optimal single-user power control algorithm, using the optimal policy as a bench mark for performance.

One obvious way to reduce the complexity of the power control algorithm is to implement a standard threshold policy, much like the policies employed in IS-95. When the SIR exceeds the set threshold, the mobile unit is instructed to power down; otherwise, the unit powers up. The particular threshold used in the algorithm may depend on the estimated channel parameters.

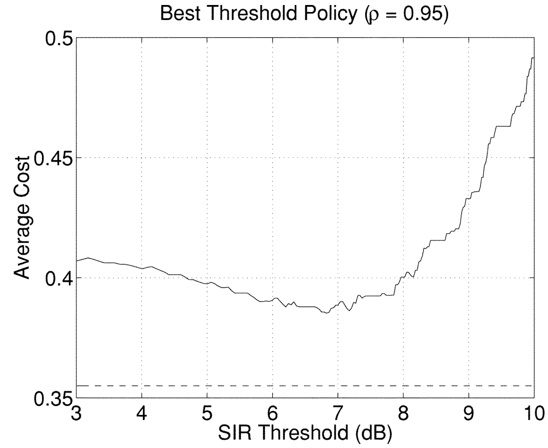

Fig. 3. Expected cost of threshold policies for various thresholds. The performance of the best threshold policy comes close to that of the optimal single-user policy.

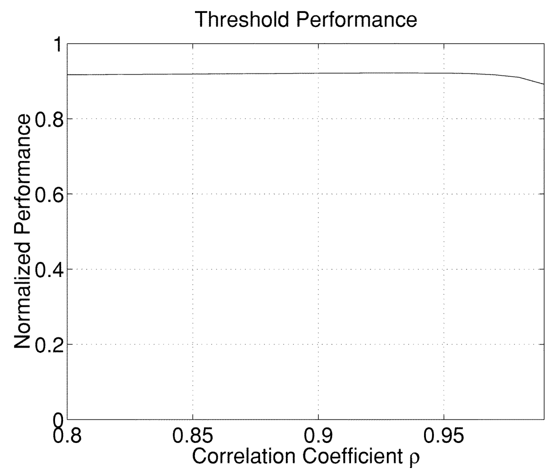

Fig. 4. Normalized performance of a threshold policy designed for a correlation coefficient of $\rho = 0.95$. The performance loss associated with this suboptimal scheme is minimal with a degradation of at most 10%.

As shown in Fig. 3, the performance of the best SIR threshold policy comes very close to that of the optimal single-user policy. An SIR threshold type of policy also seems very robust. In Fig. 4, we show the performance of a threshold policy designed for $\rho = 0.95$, and employed over channels with various correlation coefficients. The performance metric is normalized in that we present the ratio of the optimal cost over the cost incurred by the threshold policy. Performance degradation is at most 10%. Moreover, the worst performance occurs when the channel is highly correlated, and thus easily predictable. These results motivate the use of an SIR based threshold policy. The dynamic programming framework can then be used to compute the best possible threshold for any given system parameters. Alternatively, the threshold can be varied dynamically according to the current channel profile of the corresponding user.

Before we conclude this section, we provide a brief comparison of algorithms with different complexity. We mentioned previously how we expect simpler algorithms with frequent updates to outperform more complex power control algorithms. This intuition is confirmed numerically for the cost function and the wireless channel introduced in this section. Fig. 5 shows the

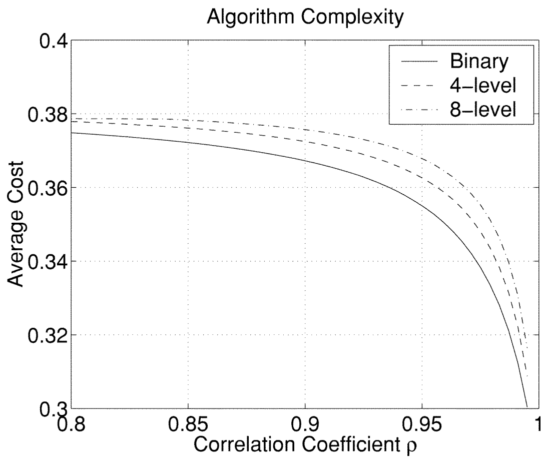

Fig. 5. Performance of algorithms with various complexity. For a fixed feedback bit rate, the optimal binary algorithms outperforms the more complex schemes.

performance of algorithms with different complexity. We emphasize that the three power control policies are compared for a fixed feedback bit rate. Thus, the binary policy adjusts the transmit power of the mobiles twice as often as the *four-level updates* policy, and three times as often as the *eight-level updates* policy. Furthermore, to insure a fair comparison, we tailored the policies to the wireless channel. That is, for any particular correlation coefficient, we compare the three best policies within their respective classes. We can see that the binary up/down algorithm outperforms more complex algorithms, corroborating our initial intuition and offering support for our detailed study of binary up/down power control algorithms as opposed to more complex schemes.

VI. Discussion and Conclusion

We presented a new approach to the design of power control algorithms for cellular CDMA systems. This technique is based on a cost minimization framework and incorporates a stochastic dynamic channel model as part of the problem definition. The stochastic channel model allows for the control policy to take into account fast variations in channel gain. The framework of dynamic programming was used to characterize the optimal power control algorithm. For large systems, the multiuser solution was shown to decouple, and effectively converge to a single-user solution in which users are controlled independently.

We have also shown how the methodology derived in this paper can be applied to a Rayleigh-fading channel model with a first-order autoregressive autocorrelation function. In a numerical study, the performance of a simple threshold policy was compared with that of the optimal single-user policy. It was found that the performance loss in adopting a threshold policy is negligible, offering support to the threshold decision rules employed in IS-95 based CDMA systems.

In formulating the power control problem, one could be tempted to minimize the sum of the transmit powers

$$Z \triangleq \sum_{k=1}^{K} p(k)$$

as well as the sum of the received powers S, leading to an alternative cost function of the form

$$\frac{1}{K}\left(\lambda_z \mathrm{E}[Z] + \lambda_s \mathrm{E}[S] + \mathrm{E}\left[\sum_{k=1}^{K} g\left(\gamma(k)\right)\right]\right). \qquad (16)$$

However, a wireless connection may be subject to very large fluctuations in channel gain and the cost function of (16) would lead to the undesirable behavior, where users are dropped very rapidly to the benefit of a smaller average transmit power. Effectively, this is equivalent to limiting the coverage of a base station. To reduce average transmit power, one should consider decreasing the radius of each cell rather than reducing the quality of service for users around the cell boundary. More base stations yield higher capacity, better coverage, and lower average transmit power for mobile users.

Avenues for further research include extending this framework to multiclass networks, where both voice users and data users must coexist. Furthermore, the performance of binary up/down control should be compared against more complex control schemes, such as iterative schemes proposed in the power control literature. A more accurate analysis would also include admission control and handoff in the problem formulation.

Appendix

Theorem 1: Suppose we fix the loading factor b. Given any $\epsilon > 0$, there exists an integer M such that $K > M$ implies the existence of a stationary Markov policy $\hat{\mu}$ for which

$$J\left(\hat{\mu}\right) < J\left(\mu^*\right) + \epsilon \qquad (17)$$

where μ^* is an optimal policy, $\hat{\mu}$ is a randomized single-user policy of the form

$$\hat{\mu}(\mathbf{x}) = (u(x(1)),\, u(x(2)),\, \ldots,\, u(x(K)))$$

and u is an admissible control policy for the single-user Markov decision process described at the beginning of Section II.

Proof: The essence of the proof is to construct a single-user policy whose performance comes arbitrarily close to being optimal. First, we derive a bound for variations in the cost function g while substituting $\mathrm{E}[S]$ for S in $\gamma(k)$.

Let $\epsilon > 0$ be given. The performance metric g is bounded, continuous, and monotone decreasing; it is therefore uniformly continuous. It follows that the function $g(\gamma(k))$ is uniformly continuous in $S \geq r(k)$ for any $N_0W/N > 0$. Then, there exits $\delta > 0$ such that $|S_1 - S_2|/N < \delta$ implies

$$\left| g\left(\frac{Nr(k)}{(1+\alpha)N_0W + S_1 - r(k)}\right) - g\left(\frac{Nr(k)}{(1+\alpha)N_0W + S_2 - r(k)}\right)\right| < \frac{\epsilon}{2} \qquad (18)$$

for all $r(k) = sp$, $(s,\, p) \in \mathcal{S} \times \mathcal{P}$. Assume now that $\delta > 0$ is given. From Corollary 1, and for a fix loading factor b, we have

$$\lim_{K\to\infty} \mathrm{Var}\left[\frac{S}{N}\right] = 0.$$

We recall from Definition 1 that, for a fixed loading factor, the expression $K \to \infty$ implies $N \to \infty$ since $b = K/N$ is fixed.

Then, there exists an integer M_1 such that for a sequence of optimal policies $\{\mu_1^*, \mu_2^*, \ldots\}$ and for all $K > M_1$

$$\mathrm{Var}\left[\frac{S}{N}\right] < \frac{\epsilon\delta^2}{2g_{\max}}$$

where $g_{\max} = \max\{|g|\}$. Under μ^*, $K > M_1$, the Chebyshev inequality asserts that

$$\mathrm{Prob}\left\{\frac{|S-\mathrm{E}[S]|}{N} \geq \delta\right\} \leq \frac{1}{\delta^2}\mathrm{Var}\left[\frac{S}{N}\right] < \frac{\epsilon}{2g_{\max}}. \quad (19)$$

Consequently, for policy μ^* with $K > M_1$, we obtain

$$\begin{aligned}
&\mathrm{E}\left[g(\gamma(k))\right]\\
&\geq \mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right.\\
&\qquad \left.-\left|g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)-g(\gamma(k))\right|\right]\\
&= \mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right]\\
&\qquad - \mathrm{E}\left[\left|g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)-g(\gamma(k))\right|\right.\\
&\qquad \left.\text{given}\left\{\frac{|S-\mathrm{E}[S]|}{N}<\delta\right\}\right]\mathrm{Prob}\left\{\frac{|S-\mathrm{E}[S]|}{N}<\delta\right\}\\
&\qquad - \mathrm{E}\left[\left|g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)-g(\gamma(k))\right|\right.\\
&\qquad \left.\text{given}\left\{\frac{|S-\mathrm{E}[S]|}{N}\geq\delta\right\}\right]\mathrm{Prob}\left\{\frac{|S-\mathrm{E}[S]|}{N}\geq\delta\right\}\\
&\overset{(a)}{>} \mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right]\\
&\qquad - \frac{\epsilon}{2}\mathrm{Prob}\left\{\frac{|S-\mathrm{E}[S]|}{N}<\delta\right\}\\
&\qquad - g_{\max}\mathrm{Prob}\left\{\frac{|S-\mathrm{E}[S]|}{N}\geq\delta\right\}\\
&\overset{(b)}{>} \mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right]-\epsilon
\end{aligned}$$

where (a) follows from (18), under the assumption that $|S - \mathrm{E}[S]|/N < \delta$, and (b) follows from (19). Thus, for $K > M_1$, the optimal cost function satisfies

$$\begin{aligned}
&J(\mu^*)\\
&= \frac{1}{K}\left(\lambda\mathrm{E}[S]+\sum_{k=1}^{K}\mathrm{E}[g(\gamma(k))]\right)\\
&> \frac{1}{K}\left(\lambda\mathrm{E}[S]+\sum_{k=1}^{K}\mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right]\right)-\epsilon.
\end{aligned} \quad (20)$$

Next, we construct a single-user policy $\hat{\mu}$ such that $\mathrm{E}_{\hat{\mu}}[S] = \mathrm{E}_{\mu^*}[S]$. Fix integer K with $K > M_1$. Define the randomized policy u, where u operates on one user by

$$\mathrm{Prob}\left\{u(x)=a\right\} = \mathrm{Prob}\left\{\mu^*(\mathbf{x}) \in \{a\}\times\mathcal{A}\times\cdots\times\mathcal{A}\right\}.$$

Also, define the associated single-user policy $\hat{\mu}$ by

$$\hat{\mu}(\mathbf{x}) = (u(x(1)), u(x(2)), \ldots, u(x(K))).$$

By construction and since μ^* is nonselective, we obtain $\mathrm{E}_{\hat{\mu}}[S] = \mathrm{E}_{\mu^*}[S]$ and

$$\begin{aligned}
&\frac{1}{K}\left(\lambda\mathrm{E}_{\hat{\mu}}[S]+\sum_{k=1}^{K}\mathrm{E}_{\hat{\mu}}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}_{\hat{\mu}}[S]-r(k)}\right)\right]\right)\\
&= \frac{1}{K}\left(\lambda\mathrm{E}_{\mu^*}[S]+\sum_{k=1}^{K}\mathrm{E}_{\mu^*}\left[g\left(\frac{Nr}{(1+\alpha)N_0W+\mathrm{E}_{\mu^*}[S]-r(k)}\right)\right]\right).
\end{aligned} \quad (21)$$

The last step of the proof is to relate the left-hand side of (21) to $J(\hat{\mu})$. Because the transmit power space $\mathcal{P}$ is finite, the variance of the received power $r(k)$ is uniformly bounded across users. Since users are controlled independently in a single-user policy, uniform boundedness implies

$$\lim_{K\to\infty}\mathrm{Var}\left[\frac{S}{N}\right] = 0.$$

Consequently, there exists an integer M_2 such that $K > M_2$ implies

$$\mathrm{Var}\left[\frac{S}{N}\right] < \frac{\epsilon\delta^2}{2g_{\max}}$$

for any single-user policy $\hat{\mu}$. Now, we can use an argument analogous to that used to produce (20) to show that for $K > M_2$

$$\begin{aligned}
J(\hat{\mu}) &= \frac{1}{K}\left(\lambda\mathrm{E}[S]+\sum_{k=1}^{K}\mathrm{E}[g(\gamma(k))]\right)\\
&< \frac{1}{K}\left(\lambda\mathrm{E}[S]+\sum_{k=1}^{K}\mathrm{E}\left[g\left(\frac{Nr(k)}{(1+\alpha)N_0W+\mathrm{E}[S]-r(k)}\right)\right]\right)+\epsilon.
\end{aligned} \quad (22)$$

By (20)–(22), we conclude that, for $K > \max\{M_1, M_2\}$, there exists a single-user policy $\hat{\mu}$ such that

$$J(\mu^*) > J(\hat{\mu}) - 2\epsilon.$$

Noting that ϵ is arbitrary, this completes the proof of the theorem. ■

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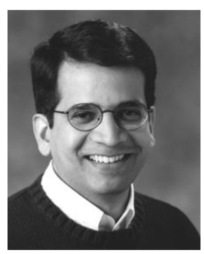

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