User Cooperation in the Absence of Phase Information at the Transmitters

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***Abstract*—In this paper, a multiuser communication system in which wireless users cooperate to transmit information to a base station is considered. The proposed scheme can significantly enlarge the achievable rate region, provided that the wireless connections between pairs of cooperating users are stronger than the connection from every user to the base station. The gains in transmission rate remain substantial even when the channel phase information is only available at the receivers, not at the transmitters. In the proposed scheme, a transmission period is divided into two time intervals. During the first time interval, wireless users send data to the base station and to the neighboring users simultaneously using a broadcast channel paradigm. During the second time interval, the users cooperate to transmit information to the base station. The achievable rate region corresponding to this paradigm is characterized under a random phase channel model for a two-user system. Results are then generalized to a multiple-user scenario. For fixed system parameters, the achievable rate region is strictly larger than that of the traditional multiple-access channel, thereby allowing a fair distribution of the wireless resources among users. Numerical analysis suggests that cooperating with a single partner is enough to achieve most of the benefits associated with cooperation.**

***Index Terms*—Achievable rate region, communication systems, dirty-paper coding (DPC), user cooperation, wireless networks.**

I. Introduction

USING multiple antennas has been shown to improve the performance of wireless communication systems in various circumstances [1]. For instance, a multiantenna configuration can be used to increase the diversity [2] or the spatial multiplexing [3] of a point-to-point wireless connection. Yet practical considerations such as small handsets and cost often limit the number of independent antennas a mobile agent can have. In such cases, it may be possible to employ user cooperation to enjoy gains comparable to those associated with multiple-input–multiple-output (MIMO) systems by forming virtual antenna arrays with the antennas of one's neighbors.

The concept of user cooperation was first introduced by Sendonaris *et al.* [4]. In a companion paper, the authors discuss implementation issues and provide a performance analysis for practical systems [5]. Virtual antenna arrays formed through user cooperation can be employed to increase the diversity or the spatial multiplexing gain of a communication system in a manner similar to a standard MIMO system. Research in this area mainly falls into three categories: designing coding techniques to improve the diversity of a system, deriving transmission strategies that increase the multiplexing gain of a system, or studying communication schemes that meet the optimal diversity-multiplexing tradeoff curve [6]. In [7], communication protocols for cooperative systems are classified into different approaches. The performance of each protocol is analyzed in terms of outage probability. This work has encouraged coding theorists to develop efficient error-control codes for user cooperation. Sophisticated coding schemes including coded cooperation [8] and space–time cooperation [9] have been proposed.

User cooperation techniques have also been employed to enlarge the achievable rate regions of wireless communication networks. One particular example where user cooperation can be used is for wireless networks where several relay nodes are incorporated in the network to help improve the transmission rates of certain users. The literature on user cooperation concepts applied to relay networks is rich. Comprehensive discussions on the subject are provided by Kramer *et al.* [10], Høst-Madsen [11], and Khojastepour *et al.* [12]. Recent results by Azarian *et al.* [13] show that the diversity-multiplexing tradeoff methodology can be extended to cooperative networks. Furthermore, the decode and forward transmission strategy achieves optimal performance.

Most of the existing work on user cooperation focuses on improving peer-to-peer link quality while considering other users as relays. In this paper, we explore the joint benefits that user cooperation may offer to all the users present in a system. Similar problems have been studied in [14] where users cooperate through finite-capacity links and later in [15] where users cooperate through orthogonal interuser channels. Herein, we characterize the achievable rate region of a user cooperation scheme applied to the uplink of wireless systems under the assumption that channel phase information is only available at the receivers, not at the transmitters. To form virtual antenna arrays, wireless users must first exchange information. As such, we propose a relatively simple scheme that divides a transmission period into two time intervals. During the first time interval, wireless users employ a broadcast paradigm to exchange information and to set up the cooperation. Each wireless user broadcasts independent information to the base station as well as to its counterparts. Once this exchange of information is completed, the system enters a virtual multiantenna mode in which the wireless users cooperate to send their information to the base station.

The remainder of this paper is organized as follows. In Section II, we present the general system model and introduce a precise formulation for the problem we wish to address.

Manuscript received June 14, 2006; revised August 29, 2007.

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Communicated by M. Médard, Associate Editor for Communications.

Color versions of Figures 4, 5, and 7–9 in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIT.2007.915711

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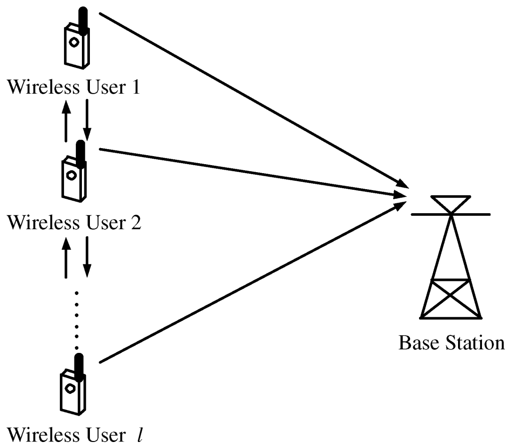

Fig. 1. User cooperation system with l users.

The derivation of the achievable rate region for the two-user system is contained in Section III. In Section IV, we extend our analysis of the achievable rate region to the three-user case. Conclusions and final remarks are presented in Section V.

II. Problem Statement

Consider a wireless communication system where l wireless users collaborate to transmit their respective information to a base station, as shown in Fig. 1. Let K_{m0} and θ_{m0} be the normalized amplitude and phase of the channel gain from user m to the base station. Similarly, let K_{mn} and θ_{mn} represent the components of the normalized channel gain from user m to user n. Our goal is to design a system where cooperation among users enables them to share system resources equitably. This is accomplished by designing a communication strategy that enlarges their achievable rate region, thereby providing users with more flexibility in choosing how to best share system resources. Since it is often difficult for a transmitter to acquire accurate channel state information (CSI), we focus on the situation where the phase information of the channel is only available at the receiver, not at the transmitter. Moreover, we impose a mean power constraint P_m on every mobile user.

The system model is expressed mathematically as follows:

$$
\begin{aligned}
Y_0(k) &= \sum_{m=1}^{l} K_{m0} e^{j\theta_{m0}(k)} X_m(k) + Z_0(k) \\
Y_n(k) &= \sum_{m=1, m\neq n}^{l} K_{mn} e^{j\theta_{mn}(k)} X_m(k) + Z_n(k)
\end{aligned} \tag{1}
$$

where k is a discrete time index. The variables $Y_0(k)$ and $Y_n(k)$ denote the signals received by the base station and user n, respectively; $X_m(k)$ represents the signal sent by wireless user m at time k. The noise sequence $\{Z_n(k)\}$ is an additive white Gaussian random process, independent of other noise processes. The phase components $\theta_{m0}(k)$ and $\theta_{mn}(k)$ are realizations of sequences of independent random variables that are not known at the transmitters. The magnitude components of the channels are assumed to be constant during the duration of the transmission, and they are known both at the transmitters and the receivers. We make the reasonable assumption that the interuser channel is symmetric, with $K_{mn} = K_{nm}$ and $\theta_{mn}(k) = \theta_{nm}(k)$. Furthermore, we assume that the channel gains corresponding to the interuser channels are greater than the gains from the users to the base station. This reflects the proximity of cooperating users. Wireless users are assumed to have the ability to transmit and receive signals at the same time and in the same frequency bands (full-duplex mode). As in [16], this implies that each wireless user knows its antenna gain and can, therefore, subtract the effects of its own signal from the information it receives. This is equivalent to assuming that there is no self-interference term in the received signal $Y_n(k)$.

In the proposed communication scheme, a transmission period is divided into two distinct time intervals. The relative proportions of these two intervals are represented by α and $(1-\alpha)$, respectively. The term α is referred to as the cooperation coefficient which controls the block length of each transmission interval. The first time interval is used to exchange the information necessary for cooperation to take place. In particular, during this interval, wireless users employ a broadcast paradigm to transmit information to their counterparts and to the base station. Conceptually, the information content of user m can be divided into l distinct parts: a first part intended for the base station, and the remaining $(l-1)$ parts intended for the other wireless users. The signal transmitted by user m during the first time interval, denoted X'_m, can therefore be written as

$$
X'_m(k) = X'_{m0}(k) + \sum_{i\neq m} X'_{mi}(k) \tag{2}
$$

where X'_{m0} contains the data intended for the base station, and X'_{mi} contains the data destined for the other users. The transmit power budget P_m of user m is split accordingly with

$$
\begin{aligned}
E[X'^2_{m0}(k)] &= \beta_m P_m \\
E[X'^2_{mi}(k)] &= P_{mi} = \beta_{mi} P_m \qquad \forall i \neq m \\
\sum_{i\neq m} E[X'^2_{mi}(k)] &= \sum_{i\neq m} P_{mi} = (1-\beta_m) P_m
\end{aligned} \tag{3}
$$

where $0 \leq \beta_m \leq 1$ is the power allocation coefficient for mobile user m. In other words, signal $X'_{m0}(k)$ carries information from user m to the base station at a rate R'_{m0} using power $\beta_m P_m$. Similarly, signal $X'_{mi}(k)$ carries information from user m to user i at a rate R'_{mi} using power $\beta_{mi} P_m$.

Inserting expressions from (2) into (1), we get

$$
\begin{aligned}
Y'_0(k) &= \sum_{m=1}^{l} K_{m0} e^{j\theta_{m0}(k)} \left(X'_{m0}(k) + \sum_{i\neq m} X'_{mi}(k) \right) + Z_0(k) \\
Y'_n(k) &= \sum_{m\neq n} K_{mn} e^{j\theta_{mn}(k)} \left(X'_{m0}(k) + \sum_{i\neq m} X'_{mi}(k) \right) + Z_n(k).
\end{aligned} \tag{4}
$$

Thus, during the first time interval, the wireless users exchange information while simultaneously transmitting data to the base station.

During the second time interval, a user may elect to transmit its own information or the information it previously received from its peers. The rate at which each of the users can transmit information is limited by the capacity of the classic multiple-access channel (MAC), as discussed in [17]. The signal received at the base station is given by

$$
Y''_0(k) = \sum_{m=1}^{l} K_{m0} e^{j\theta_{m0}(k)} X''_m(k) + Z_0(k) \tag{5}
$$

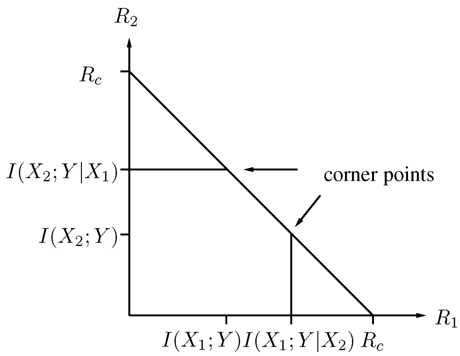

Fig. 2. Achievable rate region of two extreme cases.

where $X''_m(k)$ denotes the signal of user m during the second time interval of period k. Note that in this case signal $X''_m(k)$ is transmitted at power P_m.

III. Two-User System

In this section, we quantify the achievable rate region of the proposed cooperative scheme for the two-user case. As a preliminary step, we analyze two extreme cases that occur in the random-phase channel model introduced previously. We then proceed to derive the achievable rate regions for the two modes of operation of our system. Finally, we combine these results to get the overall achievable rate region of the proposed user cooperation scheme for the two-user system.

A. Two Extreme Cases

We identify two extreme cases: no cooperation and ideal cooperation. When there is no cooperation between users, every user sends information independently to the base station through MAC. The achievable rate region corresponding to this scheme is well known [17]. It is obtained by successive interference cancellation at the base station. The other extreme case is the idealized scenario where users cooperate to send a common message. The communication system under consideration then becomes a multiantenna 2 × 1 point-to-point system [3]. The corresponding achievable rate region is determined by the capacity of the multiple-input–single-output (MISO) system, through time sharing between the two users. The rate regions of these two extreme cases appear in Fig. 2.

As shown by Teletar [3], when phase information is not available at the transmitter and each antenna is subject to a mean power constraint, the capacity of this 2 × 1 MISO system is achieved by choosing the input vector $[X_1, X_2]$ to be a circularly symmetric complex Gaussian random variable with zero mean and covariance matrix

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}.$$

The capacity R_c of this system is given by

$$R_c = I(X_1, X_2; Y_0) = \log_2(1 + K_{10}^2 P_1 + K_{20}^2 P_2). \quad (6)$$

Note that the rate R_c can be employed to carry information from user 1 or user 2. Also note that R_c is equal to the maximum sum rate of the MAC region, as seen in Fig. 2.

The achievable rate region of a communication scheme without cooperation is a proper subset of the region corresponding to the idealized cooperative scheme. This suggests that user cooperation can significantly enlarge the achievable rate region of a multiaccess system, and motivates the user cooperation scheme introduced in Section II.

B. First Time Interval

During the first time interval, each wireless user broadcasts its information to the other user and to the base station. In this mode of operation, the communication strategy of each user reduces to a classic broadcast channel paradigm with two intended destinations. Dirty-paper coding (DPC) [18] techniques can, therefore, be employed to maximize the corresponding information throughput. There are altogether two precoding orders [19] that can be taken into account for every user. For example, user 1 can choose to encode X'_{12} first and then encode X'_{10}, or it can encode X'_{10} first and then encode X'_{12}. Because the interuser channels are assumed to be symmetric and phase information is available at the receivers, it follows that the gains of the interuser channels are known to the wireless users. On the other hand, the wireless users do not have phase information regarding their respective links to the base station. Under this assumption, the information throughput of user 1 to the base station during the first time interval R'_{10} does not change with different precoding orders. However, the interuser rate R'_{12} from user 1 to user 2 is different. When encoding X'_{12} first and then encoding X'_{10}

$$R'_{12} = \log_2\left(1 + \frac{K_{12}^2(1-\beta_1)P_1}{K_{12}^2\beta_1 P_1}\right).$$

When encoding X'_{10} first and then encoding X'_{12}

$$R'_{12} = \log_2\left(1 + K_{12}^2(1-\beta_1)P_1\right).$$

Note that $K_{12}e^{j\theta_{12}}X'_{10}$ is regarded as noncausally known interference when encoding X'_{12}. Thus, the throughput of user 1 is maximized by first encoding X'_{10}, and then encoding X'_{12}. Similarly, the throughput of user 2 is maximized by first encoding X'_{20}, and then encoding X'_{21}. For a fixed power allocation pair (β_1, β_2), the achievable rate region corresponding to the pair (R'_{10}, R'_{20}) is a typical MAC region with

$$\begin{aligned} R'_{10} &\le I(X'_{10}; Y'_0 | X'_{20}) \\ R'_{20} &\le I(X'_{20}; Y'_0 | X'_{10}) \\ R'_{10} + R'_{20} &\le I(X'_{10}, X'_{20}; Y'_0) \end{aligned} \quad (7)$$

where the defining corner points of this region are obtained using successive interference cancellation at the base station. The corner points of this polyhedron region are equal to

$$\left(\log_2\left(1 + \frac{K_{10}^2\beta_1 P_1}{1 + K_{10}^2(1-\beta_1)P_1 + K_{20}^2 P_2}\right),\right.$$
$$\left.\log_2\left(1 + \frac{K_{20}^2\beta_2 P_2}{1 + K_{10}^2(1-\beta_1)P_1 + K_{20}^2(1-\beta_2)P_2}\right)\right)$$
$$\left(\log_2\left(1 + \frac{K_{10}^2\beta_1 P_1}{1 + K_{10}^2(1-\beta_1)P_1 + K_{20}^2(1-\beta_2)P_2}\right),\right.$$
$$\left.\log_2\left(1 + \frac{K_{20}^2\beta_2 P_2}{1 + K_{10}^2 P_1 + K_{20}^2(1-\beta_2)P_2}\right)\right).$$

The interuser rates R_{12} and R_{21} are given by

$$\begin{aligned} R'_{12} &= \log_2\left(1 + K_{12}^2(1-\beta_1)P_1\right) \\ R'_{21} &= \log_2\left(1 + K_{12}^2(1-\beta_2)P_2\right). \end{aligned} \tag{8}$$

We emphasize that the wireless users exchange information with the hope that their counterparts will later transmit this information to the base station.

C. Second Time Interval

During the second time interval, the wireless users send information directly to the base station. As such, the achievable rate region becomes the classic MAC region bounded by

$$\begin{aligned} R''_{10} &\le I(X''_{10}; Y''_0 | X''_{20}) \\ R''_{20} &\le I(X''_{20}; Y''_0 | X''_{10}) \\ R''_{10} + R''_{20} &\le I(X''_{10}, X''_{20}; Y''_0) \end{aligned} \tag{9}$$

where the defining corner points are given by

$$\begin{aligned} &\left(\log_2\left(1 + \frac{K_{10}^2 P_1}{1 + K_{20}^2 P_2}\right), \log_2\left(1 + K_{20}^2 P_2\right)\right) \\ &\left(\log_2\left(1 + K_{10}^2 P_1\right), \log_2\left(1 + \frac{K_{20}^2 P_2}{1 + K_{10}^2 P_1}\right)\right). \end{aligned} \tag{10}$$

Note that cooperation among users cannot improve the sum rate in the present scenario because phase information is not available at the transmitters. However, it is possible for users to cooperate by having one user allocate part of its rate to transmitting the information it previously received from its counterpart. This cooperation opportunity is discussed in the following.

D. Achievable Rate Region

For a fixed triplet $(\alpha, \beta_1, \beta_2)$, the achievable rate region of the proposed user cooperation scheme is the union of all possible weighted combinations of rate pairs for the two modes of operation introduced previously. Mathematically, if (R'_{10}, R'_{20}) and (R''_{10}, R''_{20}) are admissible pairs for the first and second time intervals, then $(\alpha R'_{10} + (1-\alpha)R''_{10}, \alpha R'_{20} + (1-\alpha)R''_{20})$ is an achievable rate pair for the overall system. Furthermore, note that R''_{10} can carry data from user 1, or information that user 1 acquired from user 2 during the first time interval. Likewise, R''_{20} can carry data from user 2, or information that user 2 received form user 1 during the first time interval. Taking these facts into consideration, we obtain an overall rate region that is characterized by

$$\begin{aligned} R_1 &\le \alpha I(X'_{10}; Y'_0 | X'_{20}) + (1-\alpha) I(X''_{10}; Y''_0 | X''_{20}) \\ &\quad + \min(\alpha R'_{12}, (1-\alpha) I(X''_{20}; Y''_0)) \\ R_2 &\le \alpha I(X'_{20}; Y'_0 | X'_{10}) + (1-\alpha) I(X''_{20}; Y''_0 | X''_{10}) \\ &\quad + \min(\alpha R'_{21}, (1-\alpha) I(X''_{10}; Y''_0)) \\ R_1 + R_2 &\le \alpha I(X'_{10}, X'_{20}; Y'_0) + (1-\alpha) I(X''_{10}, X''_{20}; Y''_0). \end{aligned} \tag{11}$$

The term $\min(\alpha R'_{12}, (1-\alpha) I(X''_{20}; Y''_0))$ denotes the maximum amount of information user 2 can send to the base station on behalf of user 1. That is, no more than it previously received and no more than it can currently send. Similarly, $\min(\alpha R'_{21}, (1-\alpha) I(X''_{10}; Y''_0))$ denotes the maximum amount of information user 1 can send on behalf of user 2. Again, the above region is completely determined by two corner points, as shown in Fig. 3.

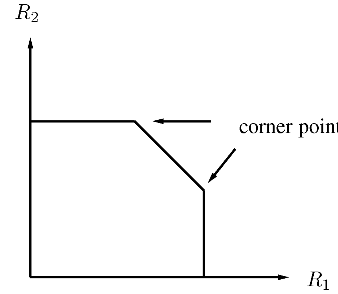

Fig. 3. Achievable rate region of user cooperation for a fixed $(\alpha, \beta_1, \beta_2)$ triplet.

Let $\mathcal{R}(\alpha, \beta_1, \beta_2)$ denote the achievable rate region corresponding to the triplet $(\alpha, \beta_1, \beta_2)$. The unconditioned achievable rate region of the proposed user-cooperation scheme is given by

$$\mathcal{R} \triangleq \bigcup_{0 \le \alpha, \beta_1, \beta_2 \le 1} \mathcal{R}(\alpha, \beta_1, \beta_2). \tag{12}$$

To characterize region $\mathcal{R}$, it suffices to identify the boundary drawn by the defining corner points of $\mathcal{R}(\alpha, \beta_1, \beta_2)$ as the parameters $(\alpha, \beta_1, \beta_2)$ vary over the unit interval. Note that the sum capacity of the communication system under consideration remains bounded by the sum capacity of the MAC over which the two users are transmitting. Also note that the noncooperative MAC region is given by $\mathcal{R}(0,1,1)$. It is, therefore, a subset of $\mathcal{R}$. Because the users do not know their respective channel gains to the base station, it is impossible to use coherent combining to improve performance [4], [16]. To characterize the boundary of $\mathcal{R}$, we can therefore assume that either R'_{12} or R'_{21} is equal to zero. We can also safely assume that

$$\begin{aligned} \alpha R'_{12} &\le (1-\alpha) I(X''_{20}; Y''_0) \\ \alpha R'_{21} &\le (1-\alpha) I(X''_{10}; Y''_0). \end{aligned} \tag{13}$$

That is, the cooperative information received by a wireless user during the first time interval should not exceed what this user can retransmit during the second interval. Energy would be squandered otherwise, with the system operating in a suboptimal fashion.

The achievable rate region of the proposed cooperative scheme is then given by the convex hull of the regions corresponding to three special cases: user 1 is helping user 2, user 2 is helping user 1, and the two users are transmitting information independently. When user 1 is helping user 2 ($\beta_1 = 1$), we have

$$\begin{aligned} R_1 &= \alpha I(X'_{10}; Y'_0) + (1-\alpha) I(X''_{10}; Y''_0) - \alpha R'_{21} \\ &= \alpha \left(I(X'_{10}; Y'_0) - R'_{21}\right) + (1-\alpha) I(X''_{10}; Y''_0) \\ &= \log_2\left(1 + K_{10}^2 P_1 + K_{20}^2 P_2\right) - \log_2\left(1 + K_{20}^2 P_2\right) \\ &\quad - \alpha \log_2\left(1 + K_{12}^2(1-\beta_2)P_2\right) \\ R_2 &= \alpha I(X'_{20}; Y'_0 | X'_{10}) + (1-\alpha) I(X''_{20}; Y''_0 | X_{10}) + \alpha R'_{21} \\ &= \alpha \left(I(X'_{20}; Y'_0 | X'_{10}) + R'_{21}\right) + (1-\alpha) I(X''_{20}; Y''_0 | X_{10}) \\ &= \log_2(1 + K_{20}^2 P_2) - \alpha \log_2\left(1 + K_{20}^2(1-\beta_2)P_2\right) \\ &\quad + \alpha \log_2\left(1 + K_{12}^2(1-\beta_2)P_2\right). \end{aligned} \tag{14}$$

Because we assume that the interuser channel is better than the uplink channels ($K_{12}^2 > K_{20}^2$), it follows that R_2 is strictly greater than $\log_2(1+K_{20}^2 P_2)$, which is the maximum achievable rate of user 2 under the traditional MAC scheme. In (14), the sum of R_1 and R_2 equals

$$\log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\alpha\log_2\left(1+K_{20}^2(1-\beta_2)P_2\right).$$

This is less than the sum-rate capacity of the MISO channel. The quantity

$$\alpha\log_2\left(1+K_{20}^2(1-\beta_2)P_2\right)$$

is the price to pay in terms of sum-rate loss for the exchange of information that take place between the users during the first time interval.

To obtain the boundary traced by the upper corner point, we maximize R_2 while keeping R_1 fixed. Let C_1 be a positive constant. Suppose that the collection of pairs of the form (α, β_2) for which

$$C_1=\alpha\log_2\left(1+K_{12}^2(1-\beta_2)P_2\right) \tag{15}$$

is nonempty. We emphasize that keeping C_1 constant is equivalent to fixing R_1. Then, maximizing R_2 is equivalent to minimizing

$$\alpha\log_2\left(1+K_{20}^2(1-\beta_2)P_2\right) \tag{16}$$

over $0 \leq \beta_2 \leq 1$ subject to (15). The constraint in (15) bounds β_2 away from one, which insures that $\log_2\left(1+K_{12}^2(1-\beta_2)P_2\right)$ is nonzero. When substituting α in (16) using the constraint of (15), we see that maximizing R_2 is equivalent to minimizing $G(\beta_2)$ where

$$G(\beta_2)=C_1\cdot\frac{\log_2\left(1+K_{20}^2(1-\beta_2)P_2\right)}{\log_2\left(1+K_{12}^2(1-\beta_2)P_2\right)}. \tag{17}$$

Because $K_{12}^2 > K_{20}^2$, it can be shown that $G(\beta_2)$ is a monotonic decreasing function of β_2 (see Appendix A). Thus, for all (α, β_2) pairs satisfying the constraint in (15), $G(\beta_2)$ is minimized when β_2 is maximized. Furthermore, from (15), we find that α is maximized when β_2 is maximized. Thus, the boundary of the corner point is determined by maximizing α. Taking (13) into consideration, the domain of α is found to be $0 \leq \alpha \leq \alpha_o'$, where (18), shown at the bottom of the page, holds. This implies that $\alpha = \alpha_o'$ gives the boundary drawn by the corner point of the achievable rate region.

Conversely, if user 2 is helping user 1 improve its rate ($\beta_2 = 1$), then the boundary drawn by the corner points is given by

$$\begin{aligned}R_1 &= \log_2(1+K_{10}^2P_1)-\alpha\log_2\left(1+K_{10}^2(1-\beta_1)P_1\right)\\ &\quad+\alpha\log_2\left(1+K_{12}^2(1-\beta_1)P_1\right)\\ R_2 &= \log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\log_2\left(1+K_{10}^2P_1\right)\\ &\quad-\alpha\log_2\left(1+K_{12}^2(1-\beta_1)P_1\right).\end{aligned} \tag{19}$$

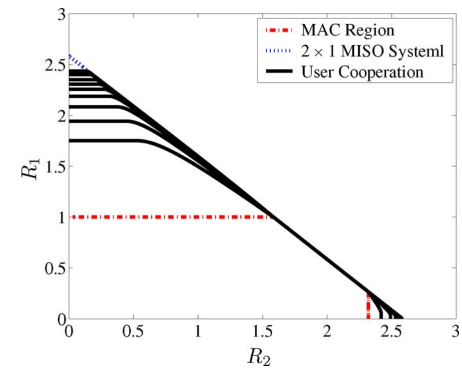

Fig. 4. Achievable rate region of the user cooperation scheme where $P_1 = P_2 = 1$, $K_{10}^2 = 4$, $K_{20}^2 = 1$, and $K_{12}^2 = 9, 16, 25, \ldots$.

In this case, $\alpha\log_2\left(1+K_{10}^2(1-\beta_1)P_1\right)$ is the price to pay for cooperation in terms of sum-rate loss. Going through a similar argument, we find that $\alpha = \alpha_o''$ where (20), shown at the bottom of the page, yields the boundary drawn by the corner points of the achievable rate region.

These results are easier to understand through an example. If the uplink channel gains are $K_{10}^2 = 4$ and $K_{20}^2 = 1$ and the interuser channel gain increases from 9 to infinity, then the achievable rate region of the proposed cooperative scheme approaches the idealized region of a MISO system. This region appears in Fig. 4. User cooperation provides a significant improvement over the traditional MAC region.

We emphasize that, in characterizing the boundary of the achievable rate region for the proposed two-user cooperation system, it is sufficient to characterize the rate regions of the special cases where only one user is being helped at a time. The overall achievable rate region then becomes the convex hull of the rate regions for these two special cases together with the traditional noncooperative MAC region.

IV. Three-User System

In this section, we extend the user cooperation scheme presented in Section III to the three-user scenario. Paralleling the development of the previous section, we first derive the achievable rate regions for the two modes of operation of the system. We then combine these regions together to identify the achievable rate region of the overall system. For convenience, we assume that at most one user is broadcasting its information at a

$$\alpha_o'=\frac{I(X_{10}'';Y_0'')}{R_{21}'+I(X_{10}'';Y_0'')}=\frac{\log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\log_2\left(1+K_{20}^2P_2\right)}{\log_2\left(1+K_{12}^2(1-\beta_2)P_2\right)+\log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\log_2\left(1+K_{20}^2P_2\right)}. \tag{18}$$

$$\alpha_o''=\frac{I(X_{20}'';Y_0'')}{R_{12}'+I(X_{20}'';Y_0'')}=\frac{\log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\log_2\left(1+K_{10}^2P_1\right)}{\log_2\left(1+K_{12}^2(1-\beta_1)P_1\right)+\log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right)-\log_2\left(1+K_{10}^2P_1\right)} \tag{20}$$

time during the first time interval. This assumption greatly simplifies the user cooperation scheme and makes the derivation of the achievable rate region for the three-user system tractable.

A. First Time Interval

During the first time interval, at most one user is broadcasting its information at a time. The remaining users send their data directly to the base station. We denote the user that is being helped by variable i. That is, for a fixed power allocation coefficient β_i, user i transmits information X'_{i0} with power $\beta_i P_i$ to the base station and sends information $\sum_{n\neq i} X'_{in}$ to the other users with power $(1-\beta_i)P_i$. The remaining users only transmit data directly to the base station. Because the phase information is only available at the receiver, the throughput of user i is maximized by first encoding X'_{i0} and then encoding the information intended for the other users. The signal received at the base station is given by (4). When decoding messages X'_{10}, X'_{20}, and X'_{30}, the base station treats $K_{i0}e^{j\theta_{i0}(k)}\left(\sum_{n\neq i} X'_{in}\right) + Z_0$ as additive noise. For a fixed β_i, the achievable rate region for $(R'_{10}, R'_{20}, R'_{30})$ is a polyhedron region

$$\mathcal{C}'_{\mathrm{MAC},i}(\mathbf{P};\mathbf{K}) = \left\{\mathcal{R} : \sum_{n\in\mathcal{S}} R'_{n0} \le \log_2\left(1 + \frac{1}{\sigma_1^2}\sum_{n\in\mathcal{S}} K_{n0}^2\beta_n P_n\right), \forall \mathcal{S} \subseteq \{1,2,3\}\right\} \quad (21)$$

where $\beta_n = 1$ for $n \neq i$, $\mathbf{P} = (P_1, P_2, P_3)$, $\mathbf{K} = (K_{10}, K_{20}, K_{30})$, and $\sigma_1^2 = 1 + K_{i0}^2(1-\beta_i)P_i$.

Because all the users can serve as receivers on their interuser channels, they can acquire the phase information of their respective channels. In addition, we assume that each user has side information about the gain of the channel linking the other two users. When user 1 is broadcasting its information, the signal-to-noise and interference ratios (SNIR) at the receivers of users 2 and 3 are known to user 1. Let

$$\widetilde{K}_{12} = \frac{K_{12}^2}{1 + K_{23}^2 P_3}$$
$$\widetilde{K}_{13} = \frac{K_{13}^2}{1 + K_{23}^2 P_2}$$

be the normalized channel gains from user 1 to users 2 and 3, respectively. Similarly, define the normalized channel gains $\widetilde{K}_{21}$, $\widetilde{K}_{23}$, $\widetilde{K}_{31}$, and $\widetilde{K}_{32}$. Under the assumption that at most one user is broadcasting at a time, the achievable rate region of the interuser subsystem is given by the achievable rate region of the broadcast channel with a total power constraint of $(1-\beta_i)P_i$.

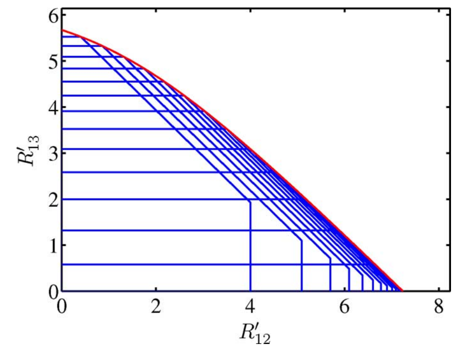

Fig. 5. Achievable rate region for (R'_{12}, R'_{13}) for a three user system when $\widetilde{K}_{12} > \widetilde{K}_{13}$.

We can rewrite this region explicitly using the broadcast and multiple-access duality [20] as

$$\begin{aligned}\mathcal{C}'_{\mathrm{BC}}\left(\beta_i, P_i; \widetilde{\mathbf{K}}_{\mathbf{i}}\right) &= \left\{\mathcal{R} : R'_{in} \le \log_2\left(1 + \frac{\widetilde{K}_{in}\beta_{in}P_i}{1 + \widetilde{K}_{in}P_i \sum_{j=1,j\neq i}^{3} \beta_{ij}\mathbf{1}_{\left[\widetilde{K}_{ij} > \widetilde{K}_{in}\right]}}\right), n \neq i\right\} \\ &= \bigcup_{\sum_{n\neq i} P_{in} = (1-\beta_i)P_i} \mathcal{C}_{\mathrm{MAC}}\left(\mathbf{P_i}; \widetilde{\mathbf{K}}_{\mathbf{i}}\right) \end{aligned} \quad (22)$$

where $\mathbf{P_i} = \{\beta_{in}P_i\}_{n=1,n\neq i}^3$, $\widetilde{\mathbf{K}}_{\mathbf{i}} = \{\widetilde{K}_{in}\}_{n=1,n\neq i}^3$, and $\mathbf{1}_{[\cdot]}$ is the indicator function. For instance, if user 1 is broadcasting information to both users 2 and 3, the achievable rate region for the interuser rate pair (R'_{12}, R'_{13}) becomes the capacity region of the broadcast channel with a fixed total power constraint $(1-\beta_1)P_1$. The boundary of this region is shown in Fig. 5.

B. Second Time Interval

During the second time interval, every user sends information directly to the base station. As such, the achievable rate region of $(R''_{10}, R''_{20}, R''_{30})$ is the typical MAC region

$$\mathcal{C}''_{\mathrm{MAC}}(\mathbf{P};\mathbf{K}) = \left\{\mathcal{R} : \sum_{m\in\mathcal{S}} R''_{m0} \le \log_2\left(1 + \sum_{m\in\mathcal{S}} K_{m0}^2 P_m\right), \forall \mathcal{S} \subseteq \{1,2,3\}\right\}. \quad (23)$$

Because users do not have the phase information associated with their links to the base station, cooperation during the second time interval cannot increase the maximum sum rate of the system. Yet, as in the two-user case, each user can allocate part of its rate to relaying data it received from the other users during the previous time interval, thereby allowing a fair repartition of the system resources.

C. Achievable Rate Region: $\widetilde{K}_{12} > \widetilde{K}_{13}$

The achievable rate region of the proposed user cooperation scheme is a constrained weighted sum of the admissible rate vectors defined by (21) and (23). For a fixed cooperation coefficient α and power allocation vectors $\vec{\beta_m} = (\beta_{mn})_{n=1,n\neq m}^{3}$, the achievable rate region of the proposed user cooperation scheme is a polyhedron region

$$\begin{aligned}\mathcal{R}\left(\alpha, \vec{\beta_1}, \vec{\beta_2}, \vec{\beta_3}\right) &= \Bigg\{ (R_{10}, R_{20}, R_{30}) : \sum_{m\in\mathcal{S}} R_{m0} \\ &\leq \alpha \log_2\left(1 + \frac{1}{\sigma_2^2}\sum_{m\in\mathcal{S}} K_{m0}^2 \beta_m P_m\right) \\ &+ (1-\alpha)\log_2\left(1 + \sum_{m\in\mathcal{S}} K_{m0}^2 P_m\right) \\ &+ \sum_{m\in\mathcal{S}, n\in\mathcal{S}^c} \min\left(\alpha R'_{mn}, (1-\alpha) R''_{n0}\right), \\ &\forall \mathcal{S} \subseteq \{1,2,3\} \Bigg\} \end{aligned} \tag{24}$$

where $\sigma_2^2 = 1 + \sum_{m=1}^{3} K_{m0}^2 (1-\beta_m) P_m$ and

$$\sum_{m\in\mathcal{S}, n\in\mathcal{S}^c} \min\left(\alpha R'_{mn}, (1-\alpha) R''_{n0}\right)$$

is maximized over all possible successive interference cancellation orders during the second time interval. The achievable rate region of the corresponding user cooperation scheme is given by

$$\mathcal{R} \triangleq co\left(\bigcup_{0\leq\alpha\leq 1, \sum_{n\neq m}\beta_{mn} = 1-\beta_m} \mathcal{R}\left(\alpha, \vec{\beta_1}, \vec{\beta_2}, \vec{\beta_3}\right)\right) \tag{25}$$

where $co(\mathcal{S})$ denotes the convex hull of the set $\mathcal{S}$. For fixed parameters, the boundary of the polyhedron region of (24) is characterized by its corner points. To fully determine $\mathcal{R}$, it suffices to draw the boundary defined by these corner points as a function of the parameters: α, $\vec{\beta_1}$, $\vec{\beta_2}$, and $\vec{\beta_3}$. First, we identify the achievable rate region for the special case where user i is being helped by the other users, as in Section III. Then, we consider the time sharing between the regions associated with these three special cases. The resulting region is optimal under the assumption that at most one user is broadcasting its information at a time. $\mathcal{R}$ is the convex hull of the rate regions of the three special cases together with the noncooperative MAC region.

Suppose user 1 is being helped by the remaining two users, i.e., $\beta_2 = \beta_3 = 1$. To maximize the throughput of user 1 in (24), we maximize R'_{10} and R''_{10}. Note that X'_{10} and X''_{10} should be decoded last at the base station to take advantage of successive interference cancellation. The achievable rate region of this special case during the first time interval is found to be

$$\begin{aligned} R'_{10} &= \log_2\left(1 + \frac{1}{\sigma_o^2} K_{10}^2 \beta_1 P_1\right) \\ \sum_{n\in\mathcal{S}} R'_{n0} &\leq \log_2\left(1 + \frac{1}{\sigma_o^2}\left(K_{10}^2\beta_1 P_1 + \sum_{n\in\mathcal{S}} K_{n0}^2 P_n\right)\right) \\ &\quad - \log_2\left(1 + \frac{1}{\sigma_o^2} K_{10}^2 \beta_1 P_1\right) \qquad \forall \mathcal{S} \subseteq \{2,3\} \end{aligned} \tag{26}$$

where $\sigma_o^2 = 1 + K_{10}^2(1-\beta_1)P_1$. In the second time interval, this region becomes

$$\begin{aligned} R''_{10} &= \log_2\left(1 + K_{10}^2 P_1\right) \\ \sum_{n\in\mathcal{S}} R''_{n0} &\leq \log_2\left(1 + K_{10}^2 P_1 + \sum_{n\in\mathcal{S}} K_{n0}^2 P_n\right) \\ &\quad - \log_2\left(1 + K_{10}^2 P_1\right) \qquad \forall \mathcal{S} \subseteq \{2,3\}. \end{aligned} \tag{27}$$

Combining the two regions given by (26) and (27), we obtain an overall achievable rate region for this specific case

$$\begin{aligned} R_{10} &= \alpha R'_{10} + (1-\alpha) R''_{10} + \sum_{n=2}^{3} \min\left(\alpha R'_{1n}, (1-\alpha) R''_{n0}\right) \\ \sum_{n\in\mathcal{S}} R_{n0} &\leq \alpha \sum_{n\in\mathcal{S}} R'_{n0} + (1-\alpha)\sum_{n\in\mathcal{S}} R''_{n0} \\ &\quad - \sum_{n\in\mathcal{S}} \min\left(\alpha R'_{1n}, (1-\alpha) R''_{n0}\right) \qquad \forall \mathcal{S} \subseteq \{2,3\}. \end{aligned} \tag{28}$$

We denote the effective interuser rates from user 1 to the remaining users by

$$R_1^e = \sum_{n=2}^{3} \min\left(\alpha R'_{1n}, (1-\alpha) R''_{n0}\right).$$

As in the two-user case, we can assume without loss of generality that

$$\alpha R'_{1n} \leq (1-\alpha) R''_{n0} \qquad \forall n \in \{2,3\}$$

because otherwise system resources are being wasted. The variable α is, therefore, constrained by

$$0 \leq \alpha \leq \min\left(\frac{R''_{20}}{R'_{12} + R''_{20}}, \frac{R''_{30}}{R'_{13} + R''_{30}}\right). \tag{29}$$

Under this condition, $R_1^e = \sum_{n=2}^{3} \alpha R'_{1n} = \alpha R'_{12} + \alpha R'_{13}$, which can be interpreted as α times the sum rate of the broadcast channel from user 1 to the rest of the users. We can rewrite the achievable rate region for this special case as

$$\begin{aligned} R_{10} &= \alpha R'_{10} + (1-\alpha) R''_{10} + \alpha\left(R'_{12} + R'_{13}\right) \\ \sum_{n\in\mathcal{S}} R_{n0} &\leq \alpha \sum_{n\in\mathcal{S}} R'_{n0} + (1-\alpha)\sum_{n\in\mathcal{S}} R''_{n0} - \alpha \sum_{n\in\mathcal{S}} R'_{1n}, \\ &\qquad \forall \mathcal{S} \subseteq \{2,3\}. \end{aligned} \tag{30}$$

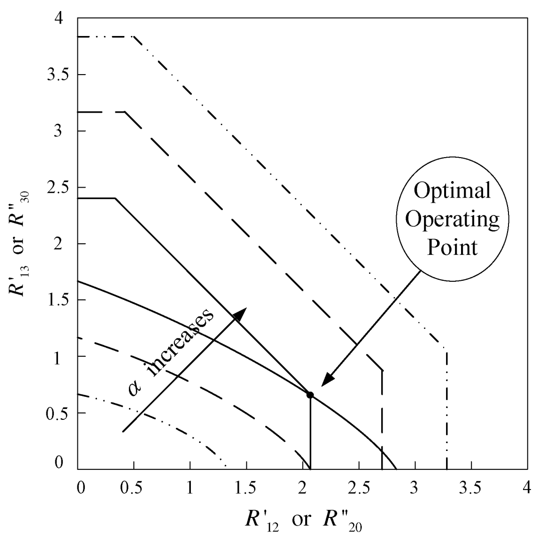

Fig. 6. Achievable rate regions for different α.

The boundary of the achievable rate region specified in (30) can be obtained by maximizing R_{10} while keeping (R_{20}, R_{30}) fixed. As a natural generalization of the results obtained in the two-user case, we can show that the boundary of the corner points is obtained by maximizing α (see Appendix B). The optimal system parameters α, β_{12}, and β_{13} should, therefore, satisfy the following condition:

$$\alpha = \frac{R''_{20}}{R'_{12} + R''_{20}} = \frac{R''_{30}}{R'_{13} + R''_{30}}. \quad (31)$$

The intuition behind the optimal conditions (31) can be visualized by comparing the rate regions described in (22) and (27). For fixed $\widetilde{\mathbf{K}}_{\mathbf{1}}$ and β_1, we can vary α until the boundaries of the two achievable rate regions meet. Note that when α is small, the interuser rate region defined in (22) is always included in the rate region of (27). As we increase α, one region is shrinking, while the other is expanding. The optimal operating point occurs when the two boundaries of the sum rate first meet each other. This is illustrated in Fig. 6. The meeting point in the MAC region shown in (27) identifies the order of the successive interference cancellation technique to be employed at the base station during the second time interval. Without loss of generality, we assume that $\widetilde{K}_{12} \geq \widetilde{K}_{13}$, which implies $\max R'_{12} \geq \max R'_{13}$. Under this assumption, the optimal operating point is found to be

$$R''_{n0} = \log_2\left(1 + \sum_{i=1}^{n} K_{i0}^2 P_i\right) - \log_2\left(1 + \sum_{i=1}^{n-1} K_{i0}^2 P_i\right), \quad n = 2, 3.$$

This result asserts that user 1 should cooperate first with the user which has the strongest interuser channel, which is user 2 in our case. Furthermore, user 1 should send data at the maximum rate user 2 can support during the second time interval. Excess data that cannot be accommodated by user 2 should then be sent to user 3. At the base station, the optimal decoding order is to first decode X''_{30}, then X''_{20}, and finally X''_{10}. The optimal α and power allocation vector β_1 satisfy the following equations:

$$(1-\alpha)\left(\log_2\left(1 + \sum_{i=1}^{n} K_{i0}^2 P_i\right) - \log_2\left(1 + \sum_{i=1}^{n-1} K_{i0}^2 P_i\right)\right) = \alpha \log_2\left(1 + \frac{\widetilde{K}_{1n}\beta_{1n}P_1}{1 + \widetilde{K}_{1n}\sum_{j=2}^{n-1}\beta_{1j}P_1}\right), \quad n = 2, 3 \quad (32)$$

which coincide with the optimal conditions obtained in (31). For β_1 fixed, we have two equations with two unknowns in (32). We can then solve for α and β_{12}. After inserting the solution into (28), the boundary of the achievable rate region for the special case where user 1 is being helped is obtained by varying β_1 from 0 to 1. For example, if $K_{10}^2 = 1, K_{20}^2 = 4, K_{30}^2 = 2, K_{12}^2 = 10, K_{13}^2 = 5$, and $K_{23}^2 = 6$, the rate region is shown in Fig. 7.

We can conduct a similar analysis for the cases where users 2 and 3 are being helped. This leads to the achievable rate regions for the corresponding systems. With the same parameters as before, the union of the achievable rate regions of the three special cases is shown in Fig. 8. The achievable rate region of the three-user cooperative system is the convex hull of the region shown in Fig. 8.

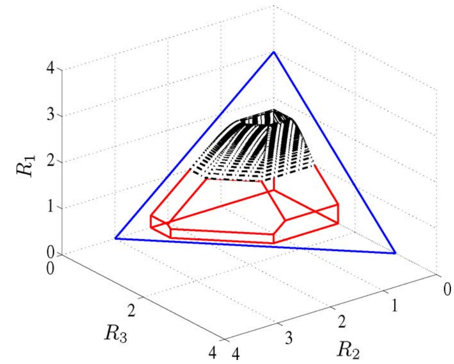

Fig. 7. Achievable rate region where users 2 and 3 are helping user 1.

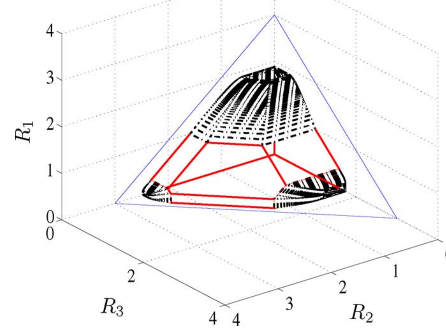

Fig. 8. Union of the achievable rate regions of the three special cases.

D. Selecting a Partner for Cooperation

Characterizing the achievable rate region for the three-user system allows us to compare the gain associated with cooperating with a single user to the gain of cooperating with multiple users. We compare these gains by projecting the three-user achievable rate region onto a 2-D rate plane. Using the example

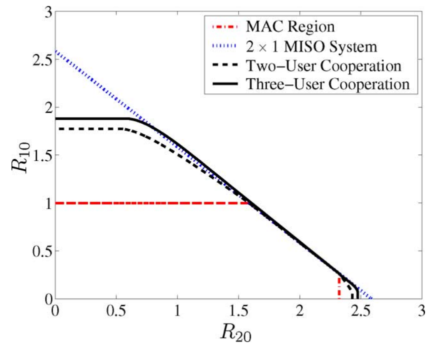

Fig. 9. Achievable rate region for users 1 and 2.

contained in Section IV-C, we plot in Fig. 9 the projection of the three-user achievable rate region onto the (R_{10}, R_{20}) plane, together with the original rate region of the two-user system (cooperation between users 1 and 2). From this graph, we can see that the benefits of cooperating with more than one user are minor for the system at hand. The projected achievable rate region for the three-user system is only slightly larger than the region for the two-user case. This suggests that user cooperation offers diminishing returns as the number of users involved increases.

The simplified scheme where a user chooses a single partner to cooperate with seems to work well, as long as this partner is selected adequately. In particular, a neighbor is a good prospect if both the interuser link and the link from this neighbor to the base station have large gains. The additional benefits of cooperating with a second neighbor appear small, in part due to the order of the successive interference cancellation that takes place at the base station. The message of this second cooperative neighbor is decoded first, which implies that the other two received signals are treated as noise during the decoding process. Because these latter two signals have substantial power, the potential benefits of cooperating with an additional neighbor are restricted by the interuser interference present at the base station. This explains the small difference between the rate region associated with the two-user system and the projected rate region of the three-user system shown in Fig. 9.

V. Conclusion

In this paper, we proposed a straightforward user cooperation scheme under the assumption that the phase information of the channel is only available at the receivers, not at the transmitters. User cooperation was found to significantly increase the achievable rate region of a two-user system whenever the interuser channel is better than the channels between the users and the base station. A larger rate region offers greater flexibility in sharing system resources equitably among wireless users.

Using similar techniques, we extended our results to a three-user scenario and we derived the achievable rate region for this more elaborate system. The region is again strictly larger than the typical MAC region under the condition that the interuser channels are stronger than the channels between the users and the base station. After comparing the achievable rate region of the three-user system and that of the two-user system, it seems that performance improvements associated with cooperation offer diminishing returns as the number of users increases. Furthermore, numerical results suggest that a reasonable strategy consists in selecting a good partner and cooperating with only one user. This is encouraging as most of the gains associated with user cooperation can be realized using this relatively simple and practical strategy. We note that there is a cost in terms of sum-rate loss associated with the proposed user cooperation scheme. However, this cost is counterbalanced by the freedom to better share system resources among the wireless users present in the system. The concept of user cooperation and the exchange of information for a better utilization of system resources are important techniques that will be integrated in future communication systems.

Appendix A
Monotonicity of $G(\beta_2)$

In this appendix, we prove that the function $G(\beta_2)$ defined in Section III-D is a monotone decreasing function of β_2. For convenience, we repeat the definition of $G(\beta_2)$, which appeared in (17) as

$$G(\beta_2) \triangleq C_1 \cdot \frac{\log_2\left(1 + K_{20}^2(1-\beta_2)P_2\right)}{\log_2\left(1 + K_{12}^2(1-\beta_2)P_2\right)}.$$

Here, C_1 is a positive constant and $K_{12}^2 > K_{20}^2$. We can rewrite $G(\beta_2)$ as follows:

$$G(\beta_2) = C_1 \cdot \frac{\log_2(1+\lambda_1)}{\log_2(1+\lambda_2)}.$$

where $\lambda_1 = K_{20}^2(1-\beta_2)P_2$, $\lambda_2 = K_{12}^2(1-\beta_2)P_2$, and $\lambda_1 < \lambda_2$. To show that $G(\beta_2)$ is a monotonic decreasing function, we take the first derivative of $G(\beta_2)$ and show that it is less than zero for all β_2. The first derivative of $G(\beta_2)$ can be obtained as

$$\frac{dG(\beta_2)}{d\beta_2} = \frac{H(\beta_2)}{\left(\log_2(1+\lambda_2)\right)^2}$$

where

$$H(\beta_2) = \frac{\lambda_2(1+\lambda_1)\log_2(1+\lambda_1) - \lambda_1(1+\lambda_2)\log_2(1+\lambda_2)}{(1-\beta_2)(1+\lambda_1)(1+\lambda_2).}$$

The constraint in (15) bounds β_2 away from 1, which guarantees $\log_2(1+\lambda_2) > 0$ for all $\beta_2 \in [0,1)$. Therefore, it is sufficient to show that $H(\beta_2)$ is less than zero for all $\beta_2 \in [0,1)$. Defining $J(x) \triangleq (1+1/x)\log_2(1+x)$, we can rewrite $H(\beta_2)$ in the form of

$$H(\beta_2) = \frac{\lambda_1\lambda_2(J(\lambda_1) - J(\lambda_2))}{(1-\beta_2)(1+\lambda_1)(1+\lambda_2)} = \frac{(J(\lambda_1) - J(\lambda_2))}{C_2(\beta_2)}.$$

Because $C_2(\beta_2)$ is a positive function for all $\beta_2 \in [0,1)$, it suffices to show that $J(\lambda_1) < J(\lambda_2)$ for $\lambda_1 < \lambda_2$, i.e., $J(x)$ is a monotonic increasing function. The first derivative of $J(x)$ can be obtained as

$$\begin{aligned}\frac{dJ(x)}{dx} &= \frac{1}{1+x} + \frac{x - (1+x)\log_2(1+x)}{(1+x)x^2} \\ &= \frac{x^2 + x - (1+x)\log_2(1+x)}{(1+x)x^2} \\ &= \frac{(1+x)(x - \log_2(1+x))}{1+x} > 0 \qquad \forall x \in (0,\infty).\end{aligned}$$

Therefore, $J(\lambda_1) - J(\lambda_2) < 0$ for all $0 < \lambda_1 < \lambda_2$. For all $\beta_2 \in [0,1)$, $G'(\beta_2)$ is strictly less than zero. This proves that

$G(\beta_2)$ is a monotonic decreasing function and the minimum value is achieved when β_2 is maximized.

Appendix B
Selection of α in Three-User Case

In this appendix, we show that R_{10} is an increasing function of α when R_{20} and R_{30} are kept constant. From (30), we can obtain the detailed achievable rate region for each user

$$\begin{aligned} R_{10} &= \log_2\left(1+K_{10}^2 P_1\right) + \alpha\left(R'_{12}+R'_{13}\right) \\ &\quad - \alpha\log_2\left(1+K_{10}^2\left(1-\beta_1\right)P_1\right) \\ R_{20} &\le \log_2\left(1+K_{10}^2P_1+K_{20}^2P_2\right) - \alpha R'_{12} \\ &\quad - \log_2\left(1+K_{10}^2P_1\right) \\ R_{30} &\le \log_2\left(1+K_{10}^2P_1+K_{30}^2P_3\right) - \alpha R'_{13} \\ &\quad - \log_2\left(1+K_{10}^2P_1\right) \\ R_{20}+R_{30} &\le \log_2\left(1+K_{10}^2P_1+K_{20}^2P_2+K_{30}^2P_3\right) \\ &\quad - \log_2\left(1+K_{10}^2P_1\right) - \alpha R'_{13} - \alpha R'_{12}. \end{aligned}$$

Assume that there exists a triplet $(\alpha, \beta_{12}, \beta'_{13})$ such that

$$\begin{aligned} \alpha\log_2\left(1+K'_{12}\beta_{12}P_1\right) &= C_2 \\ \alpha\log_2\left(1+K'_{13}\beta_{13}P_1\right) &= C_3 \end{aligned} \tag{33}$$

where C_2 and C_3 are positive constants. Note that these two equations automatically guarantee that the boundary of (R_{20}, R_{30}) is fixed. Maximizing R_{10} is equivalent to minimizing

$$L(\alpha) = \alpha\log_2\left(1+K_{10}^2\left(\beta_{12}+\beta_{13}\right)P_1\right)$$

over $0 \le \alpha, \beta_{12}, \beta_{13} \le 1$. Considering the constraints in (33), $L(\alpha)$ can be rewritten as

$$L(\alpha) = \alpha\log_2\left(1+\gamma_2\left(2^{\frac{C_2}{\alpha}}-1\right)+\gamma_3\left(2^{\frac{C_3}{\alpha}}-1\right)\right)$$

where $\gamma_2 = K_{10}^2/K'_{12}$ and $\gamma_3 = K_{10}^2/K'_{13}$. We can show that $L(\alpha)$ is a monotone decreasing function of α by taking the first derivative of $L(\alpha)$

$$\begin{aligned} \frac{dL(\alpha)}{d\alpha} &= \log_2\left(1+\gamma_2\left(2^{b_2}-1\right)+\gamma_3\left(2^{b_3}-1\right)\right) \\ &\quad - \frac{\gamma_2 b_2 2^{b_2}+\gamma_3 b_3 2^{b_3}}{1+\gamma_2\left(2^{b_2}-1\right)+\gamma_3\left(2^{b_3}-1\right)} \end{aligned}$$

where $b_2 = C_2/\alpha \ge 0$ and $b_3 = C_3/\alpha \ge 0$. The previous equation reaches its maximum point when $b_2 = 0$ and $b_3 = 0$. This guarantees that

$$\frac{dL(\alpha)}{d\alpha} = M\left(b_2, b_3\right) < M(0,0) = 0 \qquad \forall\, b_2 > 0, b_3 > 0.$$

$L(\alpha)$ is, therefore, a monotone decreasing function and R_{10} is maximized when α is at its maximum value. Recall from (29) that, for a fixed β_1

$$0 \le \alpha \le \min\left(\frac{R''_{20}}{R'_{12}+R''_{20}}, \frac{R''_{30}}{R'_{13}+R''_{30}}\right).$$

Furthermore, R_{10} is maximized when α is maximized. We then have

$$\alpha = \max_{\beta_{12},\beta_{13}} \min\left(\frac{R''_{20}}{R'_{12}+R''_{20}}, \frac{R''_{30}}{R'_{13}+R''_{30}}\right).$$

The optimal β_{12} and β_{13} should be such that

$$\alpha = \frac{R''_{20}}{R'_{12}+R''_{20}} = \frac{R''_{30}}{R'_{13}+R''_{30}}$$

which is the condition stated in (31).

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