How Dense Should a Sensor Network Be for Detection With Correlated Observations?

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***Abstract*—A detection problem in sensor networks is considered, where the sensor nodes are placed on a line and receive partial information about their environment. The nodes transmit a summary of their observations over a noisy communication channel to a fusion center for the purpose of detection. The observations at the sensors are samples of a spatial stochastic process, which is one of two possible signals corrupted by Gaussian noise. Two cases are considered: one where the signal is deterministic under each hypothesis, and the other where the signal is a correlated Gaussian process under each hypothesis. The nodes are assumed to be subject to a power density constraint, i.e., the power per unit distance is fixed, so that the power per node decreases linearly with the node density. Under these constraints, the central question that is addressed is: how dense should the sensor array be, i.e., is it better to use a few high-cost, high-power nodes or to have many low-cost, low-power nodes? An answer to this question is obtained by resorting to an asymptotic analysis where the number of nodes is large. In this asymptotic regime, the Gärtner-Ellis theorem and similar large-deviation theory results are used to study the impact of node density on system performance. For the deterministic signal case, it is shown that performance improves monotonically with sensor density. For the stochastic signal case, a finite sensor density is shown to be optimal.**

***Index Terms*—Decentralized detection, decision-making, distributed detection, multisensor systems, sensor network, wireless sensors.**

I. Introduction

Distributed sensor systems with the capacity to collect, analyze, and transmit environmental data have the potential to enable the next revolution in information technology. The rising interest in such systems originates primarily from the low cost of emerging miniature sensing technologies, together with the wide availability of the computing resources necessary to handle complex data. This correspondence focuses on the design of wireless sensor systems in the context of signal detection. The system model considered throughout consists of a set of geographically dispersed sensor nodes along with a central entity, called fusion center. The sensor nodes gather information about the properties or the likely occurrence of an event of interest, and then relay a summary of their observations to the fusion center. In turn, the fusion center processes the received information and makes a final decision.

The literature on decentralized detection, and more recently on detection applications in wireless sensor systems, is vast. In decentralized systems, the task of the fusion center can be reduced to a classical hypothesis testing problem where the information received from the sensor nodes is viewed as a vector observation [1], [2]. Decision tests for the fusion center are therefore well-understood and can often be derived using standard techniques from statistics.

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In the canonical decentralized detection problem formulation, observations at the sensor nodes are individually compressed to one of finitely many messages prior to being sent to the fusion center. The communication channel between each sensor node and the fusion center is assumed to be noiseless and, accordingly, the transmitted information is received reliably at the fusion center. The canonical design problem therefore consists of selecting a local quantization function for every sensor node. A remarkable result in decentralized detection is the fact that threshold rules on local likelihood ratios are optimal for the class of detection problems where observations are conditionally independent, given each hypothesis [2]. This property drastically reduces the search space for an optimal collection of local quantizers and, although the resulting problem is not necessarily easy, it is amenable to analysis in many contexts [3], [4].

While most results on the topic of decentralized detection assume that observations are conditionally independent, little is known about the more general problem where observations are conditionally dependent. Different approaches have been employed to study the latter problem. Willet *et al.* [5] present a thorough analysis for the binary quantization of a pair of dependent Gaussian random variables. Their findings indicate that even in this simple setting, an optimal detector may exhibit very complicated behavior. Kam *et al.* [6] examine the structure of an optimal fusion rule for the more encompassing scenario where multiple binary sensors observe conditionally dependent random variables. Chen and Ansari [7] propose an adaptive fusion algorithm for an environment where the observations and local decisions are dependent from one sensor to another. This adaptive approach requires the knowledge of fewer system parameters. Additional studies explore the effects of correlation on the performance of distributed detection systems [8], [9].

Another common postulate in decentralized detection is the assumption that the information sent by every sensor node can be conveyed reliably to the fusion center. In the context of wireless sensor systems, this assumption of reliable communication may fail since data is transmitted over noisy channels and sensor nodes are subject to stringent power constraints. We consider an alternative framework where the fusion center only has access to a noisy version of the transmitted messages. The quality of the received information depends on the power available at the sensor nodes and on the format of the transmitted messages.

In general, wireless sensor systems offer much flexibility in their designs. A sensor network may be composed of legions of low cost sensor nodes, or it may contain only a few high-quality, high-price sensor assets. It is clear that adding sensors to an existing network can only improve overall performance. However, during the initial design phase of a system, it is natural to ask how to best allocate the available system resources. For instance, is it better to use a few high-cost, high-power nodes or to have many low-cost, low-power sensor nodes? A partial answer to this question can be found in [10], [11], where it is shown that having more low-cost sensors usually performs better provided that the sensor observations remain conditionally independent regardless of sensor density. Yet, as sensor nodes are packed more densely in a finite area, it is reasonable to expect their observations to become increasingly correlated.

In this work, we seek to provide guidelines on how dense a sensor system should be for the scenario where correlation among observations increases with sensor node density. Specifically, we consider a scenario where the sensor nodes are placed on a line, and the observations at the sensors are samples of a spatial stochastic process on the line. We assume that the nodes are subject to a power density constraint. That is, the power per unit distance is fixed, so that the power per node decreases linearly with node density. We then employ mathematical instruments from large deviation theory and statistical signal processing to derive guidelines relating sensor density, resource allocation, and overall system performance.

II. System Model

We study the basic scenario where the observed stochastic process consists of one of two possible signals corrupted by additive noise. The two possible signals, which we denote by $\mathbf{X}_0(d)$ and $\mathbf{X}_1(d)$, are assumed to be 1-dimensional Gaussian stochastic processes. As such, the finite-dimensional distributions of the two processes are determined by the expectation functions $m_j(d) = \mathrm{E}[\mathbf{X}_j(d)]$ and the covariance functions

$$\rho_j(c,d) = \mathrm{E}[(\mathbf{X}_j(c) - m_j(c))(\mathbf{X}_j(d) - m_j(d))], \quad j = 0, 1. \quad (1)$$

The observation noise process, denoted by $\mathbf{V}(d)$, is assumed to be a zero-mean stationary Gaussian process with covariance function $\rho_v(c,d)$. The continuous parameter $d \in [0,\infty)$ represents the distance from the origin on the positive real line, with the understanding that the nodes are positioned at various points along a straight line.

For a finite number of sensor nodes, let $0 \leq d_1 < \cdots < d_n < \infty$ denote the position of every node. The random variable observed at each sensor then consists of one of two possible signals corrupted by additive noise

$$Y_k = X_{j,k} + V_k, \quad k = 1, \ldots, n \quad (2)$$

where we use the convenient notation $X_{0,k} = \mathbf{X}_0(d_k)$, $X_{1,k} = \mathbf{X}_1(d_k)$, and $V_k = \mathbf{V}(d_k)$. We study the important special class of sensor nodes where each unit retransmits an amplified version of its own observation to the fusion center. This class of sensor nodes is suitable for analysis. Furthermore, it is known to perform well when the sensor observations are corrupted by additive noise and the observed signal-to-noise ratio is low [11]. In this setup, a node acts as an analog relay amplifier with a transmission function given by

$$\gamma_a(Y_k) = aY_k, \quad k = 1, \ldots, n \quad (3)$$

where a is a positive amplification factor. Messages are transmitted over wireless communication channels and the fusion center receives information U_k from sensor node k of the form

$$U_k = aY_k + W_k, \quad k = 1, \ldots, n. \quad (4)$$

In vector notation, we write

$$\underline{U} = a\underline{Y} + \underline{W} \quad (5)$$

where $\underline{U} = (U_1, \ldots, U_n)^T$ is the received information, $\underline{Y} = (Y_1, \ldots, Y_n)^T$ is a vector of spatially separated observations, and $\underline{W} = (W_1, \ldots, W_n)^T$ represents communication noise. The noise vector $\underline{W}$ is assumed to have a joint multivariate Gaussian distribution and to be independent of the observation noise vector $\underline{V}$. It follows that the received information vector $\underline{U}$ is jointly Gaussian, and thus it is characterized completely by its mean vector $\underline{m}_j = \mathrm{E}[\underline{U}|H_j]$ and its covariance matrix $\Sigma_j = \mathrm{Var}(\underline{U}|H_j)$.

The objective of the system is to decide which of the two possible signals is present. It is well-known that the class of likelihood-ratio tests, in which the normalized log-likelihood ratio is compared to a threshold, is optimal [12], [13]. A threshold decision rule on the normalized log-likelihood ratio $\mathcal{L}(\underline{u})$ is an indicator function $\mathbb{1}_{[\tau,\infty)} : \mathbf{R} \to \{0, 1\}$ where

$$\mathbb{1}_{[\tau,\infty)}(\ell) = \begin{cases} 0, & \ell \in (-\infty, \tau) \\ 1, & \ell \in [\tau, \infty) \end{cases} \quad (6)$$

with the interpretation that H_1 is accepted if $\mathbb{1}_{[\tau,\infty)}(\mathcal{L}(\underline{u})) = 1$, while H_0 is accepted otherwise. The performance of a decision test $\mathbb{1}_{[\tau,\infty)}(\cdot)$ is characterized by the error probabilities

$$\alpha = \mathrm{E}[\mathbb{1}_{[\tau,\infty)}(\mathcal{L}(\underline{U}))|H_0] \quad (7)$$
$$\beta = \mathrm{E}[\mathbb{1}_{(-\infty,\tau)}(\mathcal{L}(\underline{U}))|H_1]. \quad (8)$$

In the Neyman–Pearson problem formulation, the goal is to minimize β subject to a constraint on α. Alternatively, the Bayes problem formulation aims at minimizing the probability of error at the fusion center $P_e = \alpha P(H_0) + \beta P(H_1)$. The remainder of the current section is devoted to reviewing pertinent results from statistics and to the introduction of the necessary notation for the large-deviations analysis of the detection problem at hand. In particular, we examine two interesting special cases of the Bayesian problem. The reader is referred to Van Trees [14] and Poor [12] for a more elaborate treatment of detection theory.

A. Detection of Deterministic Signals

Consider the scenario where the two signals $\mathbf{X}_0(d)$ and $\mathbf{X}_1(d)$ are known. In this case, the covariance of $\underline{U}$ is independent of the true hypothesis and an optimal procedure for deciding between hypotheses H_0 and H_1 is a threshold test on the statistics

$$T_1(\underline{U}) \triangleq \frac{1}{n}(\underline{m}_1 - \underline{m}_0)^T \Sigma^{-1} \underline{U} \quad (9)$$

where $\Sigma = \Sigma_0 = \Sigma_1$. Note that $T_1(\underline{U})$ is a Gaussian random variable, as can be deduced from the form of (9).

Suppose that the two possible signals are given by $\mathbf{X}_1(d) = -\mathbf{X}_0(d) = m > 0$. Assume that the observation noise has covariance function $\rho_v(c,d) = \sigma^2 \rho^{|d-c|}$. Also assume that the sensor nodes are equally spaced with $d_k = d(k-1)$ for $k = 1, \ldots, n$ and $d > 0$. Let $W_1, \ldots, W_n$ be an independent sequence of random variables with marginal $\mathcal{N}(0, \sigma_w^2)$. Then, $\underline{m}_1 = -\underline{m}_0 = a(m, \ldots, m)$ and

$$\Sigma = a^2\sigma^2 \begin{bmatrix} 1 & \rho^d & \cdots & \rho^{(n-1)d} \\ \rho^d & 1 & \cdots & \rho^{(n-2)d} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{(n-1)d} & \rho^{(n-2)d} & \cdots & 1 \end{bmatrix} + \sigma_w^2 I. \quad (10)$$

It follows that the expected value of $T_1(\underline{U})$ is

$$\mathrm{E}[T_1(\underline{U})] = \mp \frac{2a^2m^2}{n} \underline{1}^T \Sigma^{-1} \underline{1} \quad (11)$$

where the leading sign is negative under hypothesis H_0 and positive under H_1. Its variance is equal to

$$\mathrm{Var}(T_1(\underline{U})) = \frac{4a^2m^2}{n^2} \underline{1}^T \Sigma^{-1} \underline{1}. \quad (12)$$

In the decision rule of (9), the covariance matrix Σ is assumed to be known at the receiver. If the two signals $\mathbf{X}_0(d)$ and $\mathbf{X}_1(d)$ are known but the noise structure and hence the covariance matrix Σ are not known, a decision can be made by applying a threshold test on the decision statistics

$$T_2(\underline{U}) \triangleq \frac{1}{n}(\underline{m}_1 - \underline{m}_0)^T \underline{U}. \tag{13}$$

Again, $T_2(\underline{U})$ is a Gaussian random variable. It is therefore determined by its mean

$$\mathrm{E}[T_2(\underline{U})] = \mp 2a^2 m^2 \tag{14}$$

with negative sign under hypothesis H_0 and positive under H_1, and variance

$$\mathrm{Var}(T_2(\underline{U})) = \frac{4a^2m^2}{n^2}\underline{1}^T \Sigma \underline{1}. \tag{15}$$

B. Detection of Stochastic Signals in Gaussian Noise

The second case we examine is the specific problem where the system attempts to detect the presence of a stochastic signal in Gaussian noise. In this case, $\mathbf{X}_0(d) = 0$ and $\mathbf{X}_1(d)$ is a zero-mean Gaussian process with covariance function $\rho_x(c,d)$. An optimal detection procedure is to apply a threshold test on the quadratic form

$$T_3(\underline{U}) = \frac{1}{n}\underline{U}^T\left(\Sigma_0^{-1} - \Sigma_1^{-1}\right)\underline{U}. \tag{16}$$

This is the well-known quadratic detector.

Suppose that $\mathbf{X}_1(d)$ is a zero-mean Gaussian process with covariance function $\rho_x(c,d) = \sigma^2 \rho^{|d-c|}$, and that the sensor nodes are equally spaced with $d_k = d(k-1)$ for $k = 1, \ldots, n$. Note that the definition of ρ above differs from its use in previous section. Here, ρ represents a measure of signal covariance. This intentional abuse of notation simplifies mathematical expressions derived in the later parts of the correspondence. Context should help prevent confusion in the meaning of ρ. Assume that $V_1, \ldots, V_n$ is an independent sequence of random variables with marginal $\mathcal{N}\left(0, \sigma_v^2\right)$. Similarly, let $W_1, \ldots, W_n$ be an independent sequence of random variables with marginal $\mathcal{N}\left(0, \sigma_w^2\right)$. Then, $\Sigma_0 = \left(a^2\sigma_v^2 + \sigma_w^2\right) I$ and

$$\Sigma_1 = a^2\sigma^2 \begin{bmatrix} 1 & \rho^d & \cdots & \rho^{(n-1)d} \\ \rho^d & 1 & \cdots & \rho^{(n-2)d} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{(n-1)d} & \rho^{(n-2)d} & \cdots & 1 \end{bmatrix} + \left(a^2\sigma_v^2 + \sigma_w^2\right) I. \tag{17}$$

The correlation between observations increases with sensor proximity. To analyze the performance of this detector, we parallel an argument presented by Poor [12] and turn to linear algebra. Since Σ_1 is symmetric and positive definite, all of its eigenvalues $\psi_1, \ldots, \psi_n$ are positive real numbers and the corresponding eigenvectors $\underline{r}_1, \ldots, \underline{r}_n$ can be chosen to be orthonormal. The covariance matrix Σ_1 and its inverse can therefore be written as

$$\Sigma_1 = \sum_{k=1}^{n} \psi_k \underline{r}_k \underline{r}_k^T \text{ and } \Sigma_1^{-1} = \sum_{k=1}^{n} \psi_k^{-1} \underline{r}_k \underline{r}_k^T. \tag{18}$$

We can rewrite (16) as

$$\begin{aligned} T_3(\underline{U}) &= \frac{1}{n}\sum_{k=1}^{n}\left(\left(a^2\sigma_v^2 + \sigma_w^2\right)^{-1} - \psi_k^{-1}\right)\underline{U}^T \underline{r}_k \underline{r}_k^T \underline{U} \\ &\triangleq \frac{1}{n}\sum_{k=1}^{n} \tilde{U}_k^2 \end{aligned} \tag{19}$$

where $\tilde{U}_1, \ldots, \tilde{U}_n$ are independent zero-mean Gaussian random variables, with variances

$$H_0 : \mathrm{Var}\left(\tilde{U}_k^2\right) = 1 - \frac{\left(a^2\sigma_v^2 + \sigma_w^2\right)}{\psi_k} \triangleq \tilde{\sigma}_{0,k}^2, \quad k = 1, \ldots, n \tag{20}$$

$$H_1 : \mathrm{Var}\left(\tilde{U}_k^2\right) = \frac{\psi_k}{(a^2\sigma_v^2 + \sigma_w^2)} - 1 \triangleq \tilde{\sigma}_{1,k}^2, \quad k = 1, \ldots, n. \tag{21}$$

A threshold test on the simpler, suboptimal statistics

$$T_4(\underline{U}) = \frac{1}{n}\underline{U}^T \underline{U} \tag{22}$$

can also be used. In this case

$$\begin{aligned} T_4(\underline{U}) &= \frac{1}{n}\underline{U}^T\left[\sum_{k=1}^{n} \underline{r}_k \underline{r}_k^T\right]\underline{U} \\ &= \frac{1}{n}\sum_{k=1}^{n} \underline{U}^T \underline{r}_k \underline{r}_k^T \underline{U} \triangleq \frac{1}{n}\sum_{k=1}^{n} \breve{U}_k^2 \end{aligned} \tag{23}$$

where $\breve{U}_1, \ldots, \breve{U}_n$ are also independent zero-mean Gaussian random variables, with variances

$$H_0 : \mathrm{Var}\left(\breve{U}_k^2\right) = \left(a^2\sigma_v^2 + \sigma_w^2\right) \triangleq \breve{\sigma}_{0,k}^2, \quad k = 1, \ldots, n \tag{24}$$

$$H_1 : \mathrm{Var}\left(\breve{U}_k^2\right) = \psi_k \triangleq \breve{\sigma}_{1,k}^2, \quad k = 1, \ldots, n. \tag{25}$$

III. Large System Analysis

A wireless sensor network may only contain a few nodes, or it may contain a large number of nodes. While it is typically easy to analyze the performance of small networks (see, e.g., [15]), evaluating the performance of larger networks is often more challenging. In this section, we introduce useful mathematical instruments from large-deviation theory. These tools will prove useful in assessing the performance of large sensor systems.

For any reasonable system, the probability of detection error at the fusion center goes to zero exponentially fast as the total number of sensor nodes contained in the system tends to infinity. It is then natural to compare different system designs based on their exponential rate of convergence to zero

$$-\lim_{n\to\infty} \frac{\log P_{e,n}}{n}. \tag{26}$$

The use of large system asymptotics yields guidelines and heuristics that can be applied to all sufficiently large systems. Moreover, design guidelines derived from such limiting regimes, where the number of sensor nodes in the system becomes large, are especially relevant in the context of sensor networks because some of these networks are envisioned to comprise thousands of nodes.

A. Large Deviation Principle

Consider a sequence of real random variables $T^{(1)}, T^{(2)}, \ldots$ where $T^{(n)}$ has probability law μ_n and logarithmic moment generating function

$$\Lambda_n(\lambda) = \log \mathrm{E}[e^{\lambda T^{(n)}}]. \tag{27}$$

The following assumption is imposed to insure that the sequence $\mu_1, \mu_2, \ldots$ satisfies the large deviation principle.

Assumption 1: For $\lambda \in \mathbf{R}$, the logarithmic moment generating function

$$\Lambda(\lambda) \triangleq \lim_{n\to\infty} \frac{1}{n}\Lambda_n(n\lambda) \tag{28}$$

exists as an extended real number. For $\mathcal{D}_\Lambda \triangleq \{\lambda \in \mathbf{R} : \Lambda(\lambda) < \infty\}$, the origin belongs to the interior of $\mathcal{D}_\Lambda$, $\Lambda(\cdot)$ is differentiable in the interior of $\mathcal{D}_\Lambda$, and $\Lambda(\cdot)$ is steep. That is

$$\lim_{\ell\to\infty} |\nabla\Lambda(\lambda_\ell)| = \infty \tag{29}$$

whenever $\{\lambda_\ell\}$ is a sequence in the interior of $\mathcal{D}_\Lambda$ converging to a boundary point.

Under Assumption 1, the large deviation principle satisfied by the sequence of measures $\mu_1, \mu_2, \ldots$ can be characterized in terms of the Fenchel–Legendre transform of $\Lambda(\lambda)$,

$$\Lambda^*(z) \triangleq \sup_{\lambda\in\mathbf{R}} \{\lambda z - \Lambda(\lambda)\}. \tag{30}$$

Theorem 1 (Gärtner–Ellis): Let Assumption 1 hold. For any closed set $F \subset \mathbf{R}$

$$\limsup_{n\to\infty} \frac{1}{n}\log\mu_n(F) \le -\inf_{z\in F}\Lambda^*(z) \tag{31}$$

and for any open set $G \subset \mathbf{R}$

$$\liminf_{n\to\infty} \frac{1}{n}\log\mu_n(G) \ge -\inf_{z\in G}\Lambda^*(z). \tag{32}$$

A detailed proof of this theorem can be found in [13].

B. Detection of Deterministic Signals

Suppose that the two possible signals are known with $\mathbf{X}_1(d) = -\mathbf{X}_0(d) = m > 0$. Assume that the sensor nodes are equally spaced with $d_k = d(k-1)$ for $k = 1, \ldots, n$. Also, assume that the observation noise has covariance function $\rho_v(c,d) = \sigma^2\rho^{|d-c|}$, and let $W_1, \ldots, W_n$ be an independent sequence of random variables with marginal $\mathcal{N}(0, \sigma_w^2)$. Then, the optimal decision procedure for choosing between hypotheses H_0 and H_1 is a threshold test on

$$T_1^{(n)}(\underline{U}_n) = \frac{1}{n}(\underline{m}_{1,n} - \underline{m}_{0,n})^T \Sigma_n^{-1}\underline{U}_n. \tag{33}$$

The corresponding logarithmic moment generating functions as defined in (28) can be shown to equal

$$\Lambda(\lambda) = \mp 2\lambda a^2 m^2\sigma_\infty^2 + 2\lambda^2 a^2 m^2 \sigma_\infty^2 \tag{34}$$

where

$$\sigma_\infty^2 \triangleq \frac{1-\rho^d}{\sigma_w^2(1-\rho^d) + a^2\sigma^2(1+\rho^d)}. \tag{35}$$

A sketch of the proof is presented in Appendix. These two logarithmic moment generating functions are obviously convex and essentially smooth, i.e., Assumption 1 holds. We conclude, by Theorem 1, that the sequence

$$T_1^{(1)}(\underline{U}_1), T_1^{(2)}(\underline{U}_2), \ldots \tag{36}$$

satisfies the large deviation principle with good rate functions

$$\begin{aligned}\Lambda^*(z) &= \sup_{\lambda\in\mathbf{R}} \left\{\lambda\left(z \pm 2a^2m^2\sigma_\infty^2\right) - 2\lambda^2 a^2 m^2 \sigma_\infty^2\right\} \\ &= \frac{\left(z \pm 2a^2m^2\sigma_\infty^2\right)^2}{8a^2m^2\sigma_\infty^2}.\end{aligned} \tag{37}$$

It follows that $\tau = 0$ achieves the best possible error exponent

$$-\lim_{n\to\infty}\frac{1}{n}\log P_e^{(n)} = \frac{a^2m^2(1-\rho^d)}{2\sigma_w^2(1-\rho^d) + 2a^2\sigma^2(1+\rho^d)}. \tag{38}$$

We can perform a similar analysis under the assumption that the noise structure is not known at the fusion center. In this case, a decision between the two hypotheses is made by comparing the empirical mean

$$T_2^{(n)}(\underline{U}_n) = \frac{1}{n}(\underline{m}_{1,n} - \underline{m}_{0,n})^T \underline{U}_n \tag{39}$$

to a threshold. The asymptotic logarithmic moment generating function is obtained by applying results on the asymptotic behavior of large matrices [16]. It is equal to

$$\Lambda(\lambda) = \mp 2\lambda a^2 m^2 + 2\lambda^2 a^2 m^2 \left(\sigma_w^2 + a^2\sigma^2\frac{1+\rho^d}{1-\rho^d}\right) \tag{40}$$

where the leading sign is negative under H_0 and positive under H_1. The sequence $T_2^{(1)}(\underline{U}_1), T_2^{(2)}(\underline{U}_2), \ldots$ satisfies the large deviation principle with good rate functions

$$\Lambda^*(z) = \frac{(z \pm 2a^2m^2)^2}{8a^2m^2\left(\sigma_w^2 + a^2\sigma^2\frac{1+\rho^d}{1-\rho^d}\right)} \tag{41}$$

and the best achievable error exponent is

$$-\lim_{n\to\infty}\frac{1}{n}\log P_e^{(n)} = \frac{a^2m^2(1-\rho^d)}{2\sigma_w^2(1-\rho^d) + 2a^2\sigma^2(1+\rho^d)}. \tag{42}$$

Surprisingly, the error exponent is the same whether the correlation structure of the noise is known at the receiver or not.

C. Detection of Stochastic Signals in Gaussian Noise

In this section, we characterize the asymptotic performance of the optimal quadratic detector of Section II-B. Again, suppose $\mathbf{X}_0(d) = 0$ and $\mathbf{X}_1(d)$ is a zero-mean Gaussian process with covariance function $\rho_x(c,d) = \sigma^2\rho^{|d-c|}$. Assume that $V_1, \ldots, V_n$ is an independent sequence of random variables with marginal $\mathcal{N}(0, \sigma_v^2)$, and let $W_1, \ldots, W_n$ be an independent sequence of random variables with marginal $\mathcal{N}(0, \sigma_w^2)$.

Using properties of Toeplitz matrices [16], we gather that under H_0 the asymptotic logarithmic moment generating function is given by (43) shown at the bottom of the page and, under H_1, by

$$\Lambda_1(\lambda) = -\frac{1}{4\pi}\int_0^{2\pi}\log\left(1 - \frac{2\lambda a^2\sigma^2\frac{1-\rho^{2d}}{1-2\rho^d\cos\omega+\rho^{2d}}}{(a^2\sigma_v^2+\sigma_w^2)}\right)d\omega. \tag{44}$$

While Theorem 1 may be used to gain insight about the form of the rate function for quadratic functionals of stationary centered Gaussian processes [17], the Gärtner–Ellis theorem is not suitable to derive a large deviation principle in such cases. The asymptotic logarithmic moment

$$\Lambda_0(\lambda) = -\frac{1}{4\pi}\int_0^{2\pi}\log\left(1 - \frac{2\lambda a^2\sigma^2\frac{1-\rho^{2d}}{1-2\rho^d\cos\omega+\rho^{2d}}}{\sigma_w^2 + a^2\sigma_v^2 + a^2\sigma^2\frac{1-\rho^{2d}}{1-2\rho^d\cos\omega+\rho^{2d}}}\right)d\omega \tag{43}$$

generating functions associated with quadratic functionals do not typically fulfill the requirements of Assumption 1. Nevertheless, analogous large-deviation results can be obtained by alternate means [18], [19]. In particular, results from Bercu *et al.* [19, Section 4.1] can be transposed to the form of the current setting to obtain a large-deviation characterization of the probabilities of error. As expected, the rate functions governing the large deviation principle of the error probabilities are given by the Fenchel–Legendre transforms of (43) and (44), respectively. The error exponent corresponding to the best threshold can therefore be computed numerically using these Fenchel-Legendre transforms.

A similar analysis can be carried out for the case where the fusion center employs a simpler, suboptimal decision rule

$$T_4^{(n)}(\underline{U}_n) = \frac{1}{n}\underline{U}_n^T\underline{U}_n. \tag{45}$$

Under hypothesis H_0, the corresponding asymptotic moment generating function is given by

$$\begin{aligned}\Lambda_0(\lambda) &= -\frac{1}{4\pi}\int_0^{2\pi}\log\left(1-2\lambda\left(a^2\sigma_v^2+\sigma_w^2\right)\right)d\omega\\ &= -\frac{1}{2}\log\left(1-2\lambda\left(a^2\sigma_v^2+\sigma_w^2\right)\right)\end{aligned} \tag{46}$$

and, under H_1, the asymptotic moment generating function becomes

$$\begin{aligned}\Lambda_1(\lambda) = -\frac{1}{4\pi}\int_0^{2\pi}\log\Bigg(1-2\lambda\Bigg(&a^2\sigma_v^2+\sigma_w^2\\ &+a^2\sigma^2\frac{1-\rho^{2d}}{1-2\rho^d\cos\omega+\rho^{2d}}\Bigg)\Bigg)d\omega.\end{aligned} \tag{47}$$

The results of Bryc and Dembo [18] can be employed to assert that the large-deviation principles on the error probabilities corresponding to this simpler case are given by the Fenchel–Legendre transforms of (46) and (47), respectively. Again, the best threshold value can be computed numerically using these Fenchel–Legendre transforms. Note that, unlike in the deterministic signal case, some knowledge about the correlation structure of the observations is needed to pick the best decision threshold.

IV. Sensor Density Analysis

Wireless sensor systems are typically subject to strict power constraints, and sensor nodes are often forced to operate on tiny energy budgets [20], [21]. It is therefore imperative to understand the interplay between system performance and resource allocation in such sensor systems. While it is clear that adding sensors to an existing network can only lower the probability of error at the fusion center, the question of how dense a sensor network should be is more difficult to answer. In other words, will a system with a few powerful nodes outperform a system with a myriad of low-power nodes? This is the question we seek to answer in this section.

For convenience and tractability, we assume uniform linear arrays of sensor nodes, i.e., $d_k = d(k-1)$ for $k = 1, \ldots, n$. The corresponding sensor density is therefore given by

$$\delta \triangleq \frac{1}{d}. \tag{48}$$

To allow a fair comparison between competing designs, we consider the specific detection problem where the various sensor network candidates offer identical coverage D and are subject to total power constraint C. More specifically, the expected consumed power summed across all nodes should not exceed C, while the length of the uniform linear array of sensors should be equal to D. The number of nodes contained in a particular system can then be computed as a function of node density, i.e., $n = \lfloor \delta D \rfloor$; and, accordingly, the expected power per node is given by

$$P \triangleq \frac{C}{n} = \frac{C}{\lfloor \delta D \rfloor}. \tag{49}$$

We emphasize that under these conditions the power budget per area is fixed, forcing a system with more powerful nodes to use fewer sensors. With this relation in mind, we present below a density analysis for the two systems introduced in the previous sections.

The behavior of a large system can be characterized by letting the total power and the area covered by the network go to infinity, with their ratio kept constant. For any reasonable system, the Bayes probability of error at the fusion center goes to zero exponentially fast as the total power C and area D become simultaneously larger. It is then natural to compare collections of systems based on their exponential rate of convergence to zero

$$-\lim_{C\to\infty}\frac{1}{C}\log P_e^{(C)}. \tag{50}$$

This type of analysis yields guidelines for the allocation of system resources in wireless sensor systems. For large area and total power, the asymptotic regime of (50) provides an adequate measure of performance; and its maximizing solution, an educated guess on how dense the network should be.

A. Detection of Deterministic Signals in Gaussian Noise

When the two possible signals are Gaussian random processes with means $\pm m$, the expected power consumed by an analog relay amplifier γ_a is given by $P = a^2m^2 + a^2\sigma^2$. From our analysis, we know that the best achievable error exponent at the fusion center for deterministic signals in Gaussian noise is equal to

$$\begin{aligned}-\lim_{C\to\infty}\frac{1}{C}\log P_e^{(C)} &= -\frac{1}{a^2m^2+a^2\sigma^2}\lim_{n\to\infty}\frac{1}{n}\log P_e^{(n)}\\ &= \frac{m^2}{2(m^2+\sigma^2)}\\ &\quad\times\frac{(1-\rho^d)}{\sigma_w^2(1-\rho^d)+a^2\sigma^2(1+\rho^d)}\end{aligned} \tag{51}$$

where the exponential rate is given by (38) and (42). Again, we emphasize that the ratio of the area covered by the network and the total power consumed by the nodes is fixed in this analysis. That is, the area and the total power increase jointly to infinity with their ratio kept constant. Note also that node density affects (51) only through the correlation coefficient ρ. It is therefore the combined effect of node density and correlation that affects the overall performance of the system. Tradeoff curves for various observation signal-to-noise ratios (SNRs) and correlation coefficients appear in Fig. 1.

It is readily seen in Fig. 1 that the error exponent increases with node density, regardless of the correlation coefficient and SNR. This can be shown rigorously. Diversity always improves performance, i.e., having more sensor nodes with each node using less power outperforms having fewer high-power nodes. The maximum error exponent is achieved in the limit where the power per node goes to zero and the node density goes to infinity

$$-\lim_{a\to 0}\lim_{C\to\infty}\frac{1}{C}\log P_e^{(C)} = \frac{-m^2\log\rho}{4\sigma^2c-2\sigma_w^2(m^2+\sigma^2)\log\rho}. \tag{52}$$

In the expression above, c denotes the specific value of the power to area ratio $(c = C/D)$ employed in the analysis.

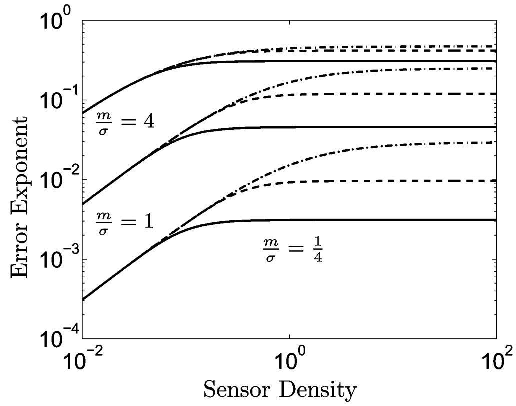

Fig. 1. Error exponent corresponding to wireless sensor nodes with analog transmission mapping $\gamma_a(Y) = aY$, radiated power $a^2(m^2+\sigma^2)$, correlation coefficient $\rho \in \{0, 0.4, 0.8\}$, and power to area ratio $c = C/D = 1$.

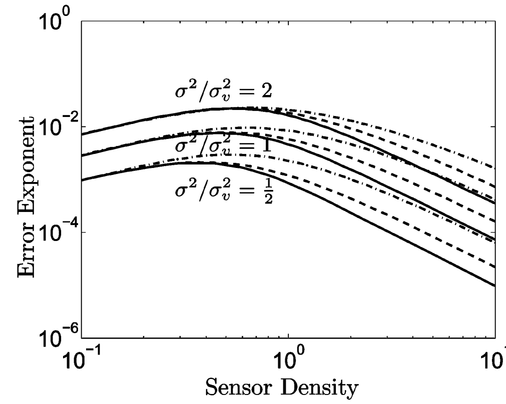

Fig. 2. Error exponent corresponding to wireless sensor nodes with optimal decision rule, for analog transmission mapping $\gamma_a(Y) = aY$, expected radiated power $a^2(\sigma_v^2 + P(H_1)\sigma^2)$, correlation coefficient $\rho \in \{0.2, 0.5, 0.8\}$, and power to area ratio $c = C/D = 1$.

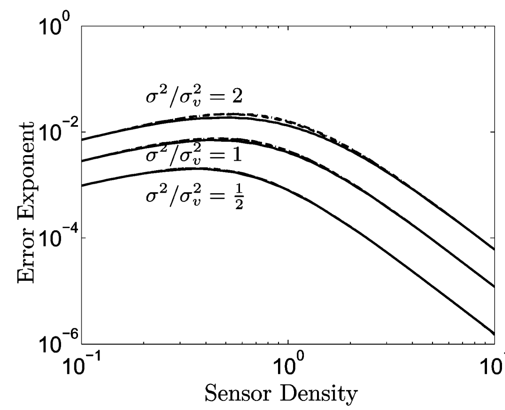

Fig. 3. Error exponent corresponding to wireless sensor nodes with simpler, suboptimal decision rule, for analog transmission mapping $\gamma_a(Y) = aY$, expected radiated power $a^2(\sigma_v^2 + P(H_1)\sigma^2)$, correlation coefficient $\rho \in \{0.2, 0.5, 0.8\}$, and power to area ratio $c = C/D = 1$.

The fact that performance always improves with node density is interesting, since although correlation and observation SNR affect overall performance, they do not change the way the sensor system should be designed. That is, systems with many low-power nodes will always perform well for the detection of deterministic signals in Gaussian noise.

B. Detection of Stochastic Signals in Gaussian Noise

Performance curves can also be obtained for the scenario where sensor nodes attempt to detect the presence of a stochastic signal in Gaussian noise. The power consumption of a relay amplifier in this context is given by

$$\begin{aligned} P &= P(H_0)a^2\sigma_v^2 + P(H_1)(a^2\sigma^2 + a^2\sigma_v^2) \\ &= a^2\sigma_v^2 + P(H_1)a^2\sigma^2. \end{aligned} \tag{53}$$

Note that the expected consumed power per sensor in this case depends on the a priori probabilities $P(H_0)$ and $P(H_1)$. For sensor density δ and power to area ratio $c = C/D$, the power per node can be computed as

$$P = \frac{c}{\delta} = a^2\sigma_v^2 + P(H_1)a^2\sigma^2. \tag{54}$$

The corresponding amplification factor a is the non-negative solution to (54). Also, the distance between adjacent sensor nodes in a linear array is given by

$$d = \frac{a^2\sigma_v^2 + P(H_1)a^2\sigma^2}{c}. \tag{55}$$

In this scenario, our results are obtained through numerical procedures because the error exponents corresponding to this setting do not admit closed-form expressions. For specific system parameters, the best achievable error exponent at the fusion center is given by

$$\begin{aligned} &\lim_{C\to\infty} \frac{1}{C}\log P_e^{(C)} \\ &\qquad = -\frac{\max\left\{\lim_{n\to\infty}\frac{1}{n}\log\alpha_n, \lim_{n\to\infty}\frac{1}{n}\log\beta_n\right\}}{a^2\sigma_v^2 + P(H_1)a^2\sigma^2} \end{aligned} \tag{56}$$

where the arguments of the maximum are given by (43) and (44) when the optimal detector is used, and by (46) and (47) when a simpler, suboptimal detector is employed instead.

Error exponents as functions of sensor density appear in Figs. 2 and 3 for the optimal and simpler detectors, respectively. The figures show rates for various observation signal-to-noise ratios and correlation coefficients. Again we emphasize that, as sensor density increases, the power per node decreases. The tradeoff between the number of observations and the quality of the information provided by each sensor node is apparent in Figs. 2 and 3. System performance improves with density up to a point where additional gains due to a larger number of samples are offset by the decay in information quality caused by diminishing power per node. The optimal operating point for a specific system corresponds to the maximum of the corresponding error exponent curve. Taking the argument of this maximization yields the optimal power per node and sensor density for the associated system.

It is interesting to note that performance improves with sensor density for the detection of deterministic signals, while a threshold effect is present for the detection of stochastic signals. This type of behavior can be explained by the form of the detector employed at the fusion center. In the case of deterministic signals, a decision is made based on a weighted sum of the received signals. As such, the mean contribution of the communication noise is zero and its variance increases proportionally to the number of sensors present in the system. On the other hand, when a quadratic detector is used, the components of the communication noise affect the mean of the decision statistics and ultimately limit the sensor density. This constitutes a fundamental distinction between the two modes of operation.

$$\Sigma_n^{-1} = \frac{1}{a^2\sigma^2}\left(\begin{bmatrix} 1 & \rho^d & \cdots & \rho^{(n-1)d} \\ \rho^d & 1 & \cdots & \rho^{(n-2)d} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{(n-1)d} & \rho^{(n-2)d} & \cdots & 1 \end{bmatrix} + \frac{\sigma_w^2}{a^2\sigma^2}I\right)^{-1}$$
$$= \frac{1}{a^2\sigma^2}\begin{bmatrix} 1 & -\rho^d & \cdots & 0 \\ -\rho^d & 1+\rho^{2d} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}\left((1-\rho^{2d})I + \frac{\sigma_w^2}{a^2\sigma^2}\begin{bmatrix} 1 & -\rho^d & \cdots & 0 \\ -\rho^d & 1+\rho^{2d} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}\right)^{-1}. \tag{58}$$

$$\underline{1}^T\Sigma_n^{-1}\underline{1} = \frac{1}{\sigma_w^2}\begin{bmatrix} 1-\rho^d \\ (1-\rho^d)^2 \\ \vdots \\ 1-\rho^d \end{bmatrix}^T\left(c\begin{bmatrix} 1 & -r & \cdots & 0 \\ -r & 1+r^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} + (\rho^d r - \rho^{2d})\left(\underline{e}_1\underline{e}_1^T + \underline{e}_n\underline{e}_n^T\right)\right)^{-1}\underline{1}. \tag{61}$$

V. Conclusion

The study of system performance as a function of sensor density allows us to explore the natural tradeoff between the total number of sensor nodes in a constrained system and the quality of the information provided by each of these nodes. Having more sensor nodes implies receiving more observations at the fusion center. On the other hand, increasing the amount of resources per node allows each wireless node to communicate its information to the fusion center more reliably. The optimal design of a constrained system therefore consists of a balance between the total number of nodes and the amount of resources per node. In this correspondence, we proposed a framework for the analysis of sensor density based on an asymptotic analysis. This framework offers guidelines on how dense a specific network should be. In particular, for a specific area and a total power budget, this framework provides an educated guess on how many nodes the system should contain, how much power each node should use, and how far apart adjacent nodes should be.

When sensor nodes are densely packed in a finite area, their observations become increasingly correlated. While conditional independence is a convenient and widely used assumption, it is likely to fail for dense networks. We showed through two examples how the theory of large deviations can be used to assess the performance of wireless sensor systems with correlated observations. In particular, we showed how the Gärtner-Ellis theorem and similar results in large-deviation theory can be employed to assess the asymptotic performance of large systems. For differentiating between known signals in Gaussian noise, the overall performance was found to improve with sensor density. Whereas for the detection of a Gaussian signal embedded in Gaussian noise, a finite sensor density was shown optimal.

The techniques presented in this correspondence can be applied in a more general setting, where system constraints and complexity are traded off against diversity and overall performance. This is very promising since the field of wireless sensor networks is relatively new, and the topic of efficient system design for dependent observations remains largely unexplored. Since these networks are envisioned to contain large numbers of nodes, our hope is that the design guidelines provided by large-deviation theory will produce good starting points for the conception and implementation of practical systems.

Appendix
Rate Function

In this section, we show that

$$\lim_{n\to\infty}\frac{\underline{1}^T\Sigma_n^{-1}\underline{1}}{n} = \frac{1-\rho^d}{\sigma_w^2(1-\rho^d)+a^2\sigma^2(1+\rho^d)}. \tag{57}$$

This equation is instrumental in finding the good rate function that governs the large deviation principle associated with the probabilities of error of an optimal detector. Note that the convergence in weak norm discussed in [16] is not sufficient to establish (57). We therefore turn to an alternative derivation and we exploit the structure of the covariance matrix Σ_n. First, we note that the inverse of Σ_n is given by (58) shown at the top of the page. Using the following substitutions:

$$r = \frac{1}{2\rho^d}\left(1+\rho^{2d}+\frac{a^2\sigma^2(1-\rho^{2d})}{\sigma_w^2}\right) - \frac{1}{2}\sqrt{\frac{1}{\rho^{2d}}\left(1+\rho^{2d}+\frac{a^2\sigma^2(1-\rho^{2d})}{\sigma_w^2}\right)^2-4} \tag{59}$$

$$c = \frac{\rho^d}{r} \tag{60}$$

we can rewrite $\underline{1}^T\Sigma_n^{-1}\underline{1}$ as shown in (61) at the top of the page. Applying the inverse formula

$$(M+xx^h)^{-1} = M^{-1} - \frac{M^{-1}xx^hM^{-1}}{1+x^hM^{-1}x} \tag{62}$$

twice, recursively, we obtain

$$\underline{1}^T\Sigma_n^{-1}\underline{1} = \frac{1}{\sigma_w^2}\begin{bmatrix} 1-\rho^d \\ (1-\rho^d)^2 \\ \vdots \\ 1-\rho^d \end{bmatrix}^T \times\frac{1}{c(1-r^2)}\begin{bmatrix} 1 & r & \cdots & r^{n-1} \\ r & 1 & \cdots & r^{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ r^{n-1} & r^{n-2} & \cdots & 1 \end{bmatrix}\underline{1} + o(n). \tag{63}$$

We can then evaluate the asymptotic value of $\frac{1}{n}\underline{1}^T\Sigma_n^{-1}\underline{1}$ explicitly, which yields

$$\lim_{n\to\infty}\frac{\underline{1}^T\Sigma_n^{-1}\underline{1}}{n} = \frac{1-\rho^d}{\sigma_w^2(1-\rho^d)+a^2\sigma^2(1+\rho^d)}. \tag{64}$$

This is precisely the statement of (57), as desired.

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Wireless Link Scheduling With Power Control and SINR Constraints

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***Abstract*—The problem of determining a minimal length schedule to satisfy given link demands in a wireless network is considered. Links are allowed to be simultaneously active if no node can simultaneously transmit and receive, no node can transmit to or receive from more than one node at a time, and a given signal-to-interference and noise ratio (SINR) is exceeded at each receiver when transmitters use optimally chosen transmit powers. We show that a) the general problem is at least as hard as the MAX-SIR-MATCHING problem, which is easier to describe and b) when the demands have a superincreasing property the problem is tractable.**

***Index Terms*—Power control, scheduling, signal-to-interference and noise ratio (SINR) constraints, wireless.**

I. Introduction

Scheduling is an access control method for the wireless medium that is particularly appealing when energy efficiency or high throughput is desired, because scheduling avoids the collisions and retransmissions of contention-based methods of medium access.

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